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Probability and Risk: Foundations and Economic Implications of Probability-Dependent Risk Preferences
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Technical Appendix

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Figures 1 and 3 in the paper illustrate probability weighting functions for two different samples drawn from the student and the general population. The experimental setup and data are extensively discussed in Bruhin, Fehr-Duda, and Epper (2010) (for the student sample) and in Epper, Fehr-Duda, and Schubert (2011) (for the representative sample). The original student data also contained observations on loss lotteries and two gain lotteries involving comparatively large outcomes. We dropped these observations in order to make the student data comparable to the representative one. Estimation results for the gain domain remain robust irrespective of whether these additional observations are included in the data or not.

The econometric models and estimation techniques are described in detail in Bruhin, Fehr-Duda, and Epper (2010) and Fehr-Duda, Bruhin, and Epper (2010). For the current study, we chose a Prelec-II specification for the probability weighting function, however. It is noteworthy that the parameter estimates and standard errors, reported in Table 3 in the paper, are based on reparameterized versions of the utility and probability weighting functions. These reparameterizations permit us to interpret the p-values as the results of a statistical test of linearity of utility and probability weights, respectively. Standard errors and confidence bands are obtained by the bootstrap method with 4000 replications (Efron and Tibshirani, 1993). We account for the fact that our data features multiple observations for each subject, i.e. we

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resample with replacement from subjects.

Complementary to the parametric estimation results, Figure 1 in the paper also shows non-parametric estimates of probability weights, obtained as follows. The estimation algorithm provides a weight for each monetary amount and each probability in the data without any assumptions of specific parametric functional forms (see Gonzalez and Wu (1999) for a similar procedure). We only assume separability of money and probability weights, and monotonicity of the valuation functional.[1] Thus, the non-parametric estimation procedure relies on very weak assumptions on subjects' preferences.

A two-outcome prospect $P = (x_1, p; x_2)$ with $x_1 > x_2 \geq 0$ is assumed to be valued as

$$V_P = \nu_{x_1}\omega_p + \nu_{x_2}(1 - \omega_p)\,, \tag{1}$$

where ν_{x_1} and ν_{x_2} denote the money weights for x_1 and x_2, and ω_p denotes the weight for probability p. Figure $A1$ shows non-parametric estimation results for the representative sample. The green dots depict the corresponding weights for each value of the prospects' arguments, x and p.

The algorithm consists of a loop, repeated iteratively until convergence. At convergence it returns estimates for the two weighting vectors ν' and ω'. The loop can be decomposed into three stages:

Stage 1 (Interpolation): Since we observe certainty equivalents ce, i.e. cash amounts, we have to transform them to the utility scale first. This is achieved by the first stage, which approximates the utility of the certainty equivalent ν_{ce} by using the current vector of money weights ν' and the observed certainty equivalent. Due to the monotonicity of the utility function and the fact that the certainty equivalent lies within the prospect's outcome range by definition, the utility of the certainty equivalent must lie between the utilities of the

[1] Monotonicity is only required for the interpolation stage described below, however. Details are available on request from the authors.

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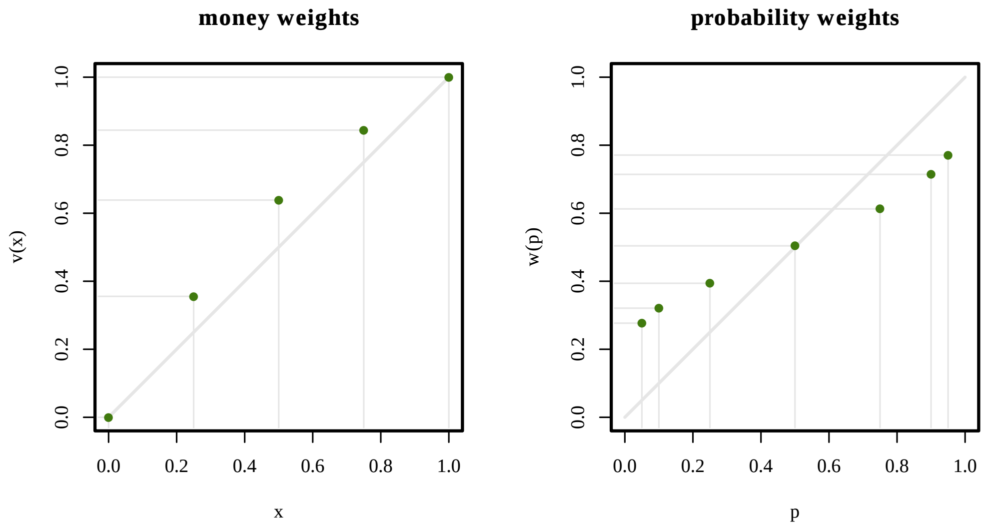
Figure A1: Non-parametric Money and Probability Weights



prospect's best (ν_{x_1}) and worst (ν_{x_2}) outcomes. Using linear interpolation,[2] we obtain money weights for the certainty equivalent by

$$\nu_{ce_i} = \nu_{x_2} + (ce_i - x_2)\frac{\nu_{x_2} - \nu_{x_1}}{x_1 - x_2}\,, \tag{2}$$

with i being an index for the observation. We assume that $\nu_{ce_i} = V_P + \varepsilon_i$ where ν_{ce_i} is the observed certainty equivalent transformed to the utility scale, V_P is the valuation of the prospect predicted at this stage, and ε_i is the residual. ε_i is assumed to be i.i.d. and symmetrical.

Stage 2 (Updating the Probability Weighting Vector): Given the results from stage 1, the corresponding weighting vector ω' is updated for each probability in the data. This is done by minimizing the sum of the weighted residuals by a Quasi-Newton method, holding the money weighting vector ν' fixed (indicated by a bar): $\bar{\nu}_{ce_i} = \bar{\nu}_{x_1}\omega_p + \bar{\nu}_{x_2}(1 - \omega_p) + \varepsilon_i$.

Stage 3 (Updating the Money Weighting Vector): This stage is equivalent to stage 2, applied to the money weighting vector instead of the probability weighting vector, i.e. we

[2]The results are robust with respect to other interpolation methods, such as cubic splines.

update ν' in $\bar{\nu}_{ce_i} = \nu_{x_1}\bar{\omega}_p + \nu_{x_2}(1-\bar{\omega}_p) + \varepsilon_i$.

Convergence is reached if the improvement in fit, evaluated by the sum of squared residuals, falls below a specified tolerance level within one iteration of the loop.

The confidence bars in Figure 1 in the paper are obtained by the bootstrap method. The method takes the panel structure of the data into account.

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