

A TALE OF TWO TAILS: On the Coexistence of Overweighting and Underweighting of Rare Extreme Events

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Abstract

Almost all important decisions in people's lives entail risky consequences. Some of these decisions involve events that materialize with a low probability but lead to extreme consequences such as loss of total wealth or accidental death. When facing such rare extreme events, people display considerable risk aversion in some situations whereas in others the opposite is the case. For example, the prospect of airplane and stock market crashes triggers high risk aversion but there is a low willingness to take out hazard or life insurance. We address this puzzle by arguing that the timing of the consequences and of uncertainty resolution are crucial for understanding these phenomena. We show that future uncertainty conjointly with people's proneness to probability distortions generates a unifying framework for explaining the coexistence of over- and underweighting of rare extreme events.

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1 Introduction

Whatever the nature of our decisions, may they concern health, wealth, love or education, hardly ever can we be sure of their outcomes. Thus, it is an important task for economists to understand, model and predict decisions under risk. However, it seems difficult to paint a coherent picture of people's risk preferences because in some situations their behaviors appear to be extremely risk averse while in others the opposite is the case. For example, many consumers purchase extended warranties for household appliances at exorbitant prices, i.e. they display extreme risk aversion in situations that involve comparatively low stakes (Cicchetti and Dubin, 1994; Huysentruyt and Read, 2010). According to the standard workhorse of economics, Expected Utility (EU) theory, consumers should be approximately risk neutral in this case (Loomes and Segal, 1994). On the other hand, many are reluctant to buy adequate life insurance, thereby exposing their loved ones to considerable poverty risk (Bernheim, Forni, Gokhale, and Kotlikoff, 2003; Cutler, Finkelstein, and McGarry, 2008). Conflicting risk attitudes are also observed in other domains of important economic decisions: Stock market participation is very low in many countries around the globe (Giannetti and Koskinen, 2010), suggesting high risk aversion, whereas inhabitants of disaster-prone areas are often not willing to take out highly subsidized insurance even though not only their wealth but also their lives are at stake (Kunreuther, 1984; Viscusi, 2010).

These disparities can be understood in terms of how tail events, i.e. rare extreme events, are evaluated. For example, paying a multiple of expected losses for extended warranties is consistent with the overweighting of an improbable appliance breakdown. Analogously, overweighting the rare event of a stock market crash makes people shy away from investing in stocks. In both cases, therefore, behavior seems to be governed by overweighting of rare extreme events. Underinsurance, as apparent in life and disaster insurance choices, however, is consistent with underweighting of rare extreme events, which raises the question how these opposite tendencies can be rationalized. As Barberis (2013a) recently noted, such an explanation is still missing.

In this paper we argue that this puzzle can be solved by accounting for two crucial characteristics of risky decision making, namely the passage of time until outcomes materialize and the process by which uncertainty resolves. Our work is inspired by mounting experimental evidence documenting that time is not an independent dimension of risky future prospects but rather interacts with risk in complex ways. In particular, there are two striking regularities. First, risk tolerance depends on the *length of delay* until outcomes materialize: It is higher for delayed outcomes than for immediate outcomes (Jones and Johnson, 1973; Shelley, 1994; Ahlbrecht and Weber, 1997; Sagristano, Trope, and Liberman, 2002; Noussair and Wu, 2006; Coble and Lusk, 2010; Abdellaoui, Diecidue, and Öncüler, 2011). Second, risk tolerance depends on the *process of uncertainty resolution*: People tend to invest less conservatively, i.e. they take on more risk, when they are informed about the outcomes of their decisions only at the end of the investment period rather than intermittently (Gneezy and Potters, 1997; Thaler, Tversky, Kahneman, and Schwartz,

1997; Gneezy, Kapteyn, and Potters, 2003; Bellemare, Krause, Kröger, and Zhang, 2005; Haigh and List, 2005). Hence, observing uncertainty resolve gradually over the course of time seems to make decision makers less risk tolerant. In which way do these experimentally discovered phenomena relate to the real-world decisions discussed above?

Delay dependence speaks directly to insurance decisions. Compare, for example, two different life insurance products, a regular life insurance policy (either a term policy or a permanent one), and a flight insurance policy that covers the same event but expires immediately after the flight. Concerning regular life insurance policies, there is little doubt about the fact that a substantial percentage of the U.S. population is underinsured (Bernheim, Forni, Gokhale, and Kotlikoff, 2003; Cutler, Finkelstein, and McGarry, 2008). However, flight insurance policies used to be extremely popular in the 1950's and 60's when they were sold at vending machines at U.S. airports.¹ Many passengers were willing to pay outrageous premiums, obviously overweighting the rare event of an imminent airplane crash (Meade, 1957).² The important difference between these two products is their maturity: A flight insurance policy is extremely short-term with uncertainty resolving within hours, whereas a regular policy extends over a much longer time horizon with uncertainty resolving at some unknown time in the (hopefully) remote future. The latter characteristic also applies to disaster insurance.

Delay dependence seems to be important in asset markets as well. Recently, the term structure of market risk premia has attracted considerable attention when assets with short maturities were found to earn much higher risk premia than assets with long maturities (van Binsbergen, Brandt, and Koijen, 2012; Andries, Eisenbach, and Schmalz, 2015; Eisenbach and Schmalz, 2016). Delay dependence of risk tolerance implies that rare adverse events that tend to be overweighted when perceived to materialize soon may end up being underweighted when expected to materialize in the remote future.

However, this reasoning begs the question why investors shy away from equities. Isn't the risk of a stock market crash a rare extreme event comparable with a natural disaster? And should we not expect high risk tolerance then? In our view, there is a crucial difference between these two types of events with respect to the process by which uncertainty resolves. Take, for example, natural disasters such as earthquakes and tsunamis. Rarely can their timing be predicted long before their actual occurrence. They literally appear out of the blue. In these cases, uncertainty resolves in one shot at some unknown time in the future. The opposite type of process is at work in the stock market. Information on asset prices is readily available, for many assets even in real time. Therefore, notwithstanding the longterm nature of many investments, uncertainty is perceived to resolve gradually over the course of time rather than in one shot at some time in the future. One can watch price bubbles building up, but not tectonic plates shifting. Process de-

¹This type of insurance seems to have been popular in other places as well. In 1997 Japanese regulatory agencies granted AIG the right to sell travel insurance via vending machines at Japanese airports, see Fingleton (2008), p. 421.

²Taken from Footnote 1 in "Air Trip Insurance", Washington and Lee Law Review 20, Issue 2, Article 16.

pendence implies that, even for long time horizons, rare adverse events tend to be overweighted when the resolution of uncertainty is observable. Our task, therefore, is to provide a unified account for the delay and process dependence of risk tolerance.

As the valuation of tail events is the central theme of this paper, a natural starting point is rank-dependent probability weighting. Rank-dependent models, such as Rank Dependent Utility Theory (RDU; Quiggin (1982)) and Cumulative Prospect Theory (CPT; Tversky and Kahneman (1992)) involve a probability weighting function that, empirically, overweights a prospect's most favorable outcome when it materializes with a small probability, but underweights it when it materializes with a large probability (for a recent review see Fehr-Duda and Epper (2012)). Such a *regressive* probability weighting function results in decision weights that overweight both tails of the outcome distribution and underweight intermediate outcomes (Wakker (2010), Chapter 6.4). Hence, it can explain why people simultaneously engage in gambling and insuring, why they favor positively skewed distributions and dislike negatively skewed ones.³ This common pattern of overweighting of both tails has an intuitive interpretation of salience: The decision makers' attention is drawn primarily to the extreme possible outcomes, termed by Lopes (1987) "the psychology of hope and fear".

However, underweighting of adverse tail events, which seems to govern disaster and life insurance choices, seems to contradict such a decision-weight based explanation.⁴ In their original paper on Prospect Theory, Kahneman and Tversky (1979) surmise that highly unlikely events are either overweighted or simply ignored because people are limited in their ability to comprehend and evaluate extreme probabilities.⁵ Of course, one could also argue that rare extreme events are underweighted because people are not aware of their existence. But many insufficiently insured people live in disaster-prone areas, even in so-called red zones (Barnes, 2011). Recently, for example, an earthquake in Amatrice, located in a notoriously earthquake-prone area of central Italy, caused 300 deaths and made many more homeless.⁶ In 2009, a similar disaster occurred in the same region only 50 km away from Amatrice. Thus, unawareness of the possibility of another earthquake in that region is a highly unlikely explanation.

Hence, underweighting of rare adverse events is to be located elsewhere: As Abdellaoui,

³Skewness preferences play an important role in finance, e.g. in rationalizing the cross-section of asset prices, the underdiversification of households, and account for other phenomena such as betting on long shots (Barberis and Huang, 2008; Snowberg and Wolfers, 2011; De Giorgi and Legg, 2012; Polkovnichenko and Zhao, 2013).

⁴The same argument applies to Friedman and Savage (1948) who proposed a utility function that accommodates the simultaneous occurrence of gambling and insuring, i.e. addressing rare positive and rare negative events. At a given wealth level, this utility function is convex with respect to wealth increases, which may lead to gambling behavior, but concave with respect to wealth decreases and, thus, predicts insurance purchases. By that same token, the model predicts risk aversion regarding *all* types of rare negative events and, therefore, cannot explain underinsurance. For a critique of their approach see also Markowitz (1952).

⁵Kahneman and Tversky (1979) go on to argue that, consequently, the probability weighting function is not well-behaved near the end-points. This argument does not appear in the cumulative version of Prospect Theory in Tversky and Kahneman (1992) any more.

⁶In fact, the earth has not stopped trembling since then.

Diecidue, and Öncüler (2011) have recently shown experimentally, the delay dependence of risk tolerance is predominantly due to a shift of the probability weighing function. Probability weights for future events tend to become more optimistic with increasing time delay, which implies that decision weights for tail events react in a predictable direction: Overweighting of rare adverse events, occurring in decisions with short time delays, becomes progressively less pronounced as the time horizon shifts out into the future. Thus, events perceived to occur in the distant future may even end up being underweighted.

Aside from regressiveness, which drives the weighting of tail events, probability weights exhibit another important characteristic: subproportionality.⁷ This feature maps the famous Allais common-ratio paradox (Allais, 1953) and induces a strong decline in risk tolerance in the case of gradual resolution of uncertainty when decision weights are compounded sequentially over short time horizons (Segal, 1987a,b, 1990). This finding is reminiscent of *myopic loss aversion* (Bernartzi and Thaler, 1995; Barberis, Huang, and Thaler, 2006), which makes people pronouncedly risk averse for short time horizons. Contrary to myopic loss aversion, *myopic probability weighting* is a phenomenon that emerges independently of the location of the reference point. The crucial question that remains is what mechanism drives the delay dependence of probability weights.

We will present a theoretical framework in which the delay dependence of probability weights is generated by the uncertainty inherent in the future. In particular, something unrelated to the prospect under consideration may go wrong before outcomes materialize which reduces the chances of actually obtaining the expected outcomes. Imagine, for example, that you are planning a vacation on the Maldives. On the day you are supposed to fly to the Maldives you find out that your passport has expired. Therefore, you will lose several days of recreation and have to incur costs for rescheduling your flight. This “something-went-wrong” event generates an outcome that is worse than the minimum level of utility you had expected from vacationing on the Maldives. Aside from forgetting to renew your passport, all kinds of adverse contingencies may arise after you have booked your vacation. For example, a serious illness in your family may occur. Therefore, it is plausible to assume that the probability of something going wrong is perceived to increase with the length of delay until outcomes materialize.

If people integrate this type of future uncertainty into their valuations of future prospects, the passage of time drives a wedge between risk tolerance with respect to immediate risks versus delayed risks. Obviously, the way in which future uncertainty affects risk tolerance depends on the characteristics of people’s risk preferences, in particular on the features of probability weighting. In the following, we show that regressiveness and subproportionality affect the decision weights of future outcomes in the desired direction. Whereas regressiveness governs the weighting of tail events, the ultimate driver of both delay dependence and process dependence is a single feature of probability weighting, subproportionality, i.e. people’s proneness to common-ratio violations.

⁷Precise definitions can be found in Section 2.

We show, first, that in the presence of inherent uncertainty, probability weights become more optimistic with increasing delay. Thereby, unfavorable tail events receive progressively less weight and may end up being underweighted altogether. The driver of this effect is that unfavorable tail events lose their salience when many things may go wrong. Second, gradual resolution of uncertainty counteracts this effect, a result of compounding subproportional decision weights.

Underinsurance with respect to natural disasters is a prevalent phenomenon. Globally, between 1960 and 2011 nearly 60% of major natural catastrophes were uninsured (von Peter, von Dahlen, and Saxena, 2012). Even in high-income countries, only 50% of the damage resulting from catastrophes, such as earthquakes, tsunamis and floods, were covered by insurance contracts. They argue that uninsured losses stemming from major natural catastrophes have large and significant negative effects on economic activity, both on impact and over the longer run. If underinsurance is driven by delay-dependent risk tolerance caused by the inherent uncertainty of the future, there is probably not much policy can do about changing preferences - that something may go wrong is a fact of life. Increasing the frequency of feedback may reduce excessive risk tolerance but may not be feasible. A similar situation arises for climate change mitigation. The upshot of these insights is that mandatory insurance and environmental taxes may be the only effective instruments to counter delay-dependent risk tolerance.

To the best of our knowledge, we are the first to present a preference-based explanation of the coexistence of overweighting and underweighting of rare extreme events, thereby providing a rationale both for the observed variations in real-world risk tolerance and a host of experimental findings.⁸ We are not the first to acknowledge that “[a]nything that is delayed is almost by definition uncertain” (Prelec and Loewenstein (1991), p.784). A large body of previous research has focused on the implications of future uncertainty for discounting behavior (Sozou, 1998; Dasgupta and Maskin, 2005; Bommier, 2006; Pennesi, 2015). Related to our approach, Halevy (2008), Walther (2010) and Saito (2015) deal with the impact of probability weighting on hyperbolic discounting. Risk preferences have mostly been studied in the context of purely atemporal settings. For example, Quiggin (2003) investigates the consequences of background risk for generalized expected utility models. Recently, interactions of time and risk were discussed by Baucells and Heukamp (2012). They present an axiomatic approach for the domain of simple prospects with only one non-zero outcome, which cannot address the question of the valuation of tail events. Regarding the sequential resolution of uncertainty, Segal examines the relationship between subproportionality and two-stage lotteries (Segal, 1987a,b, 1990). Dillenberger (2010) analyzes necessary and sufficient conditions for preferences to favor one-shot resolution of uncertainty, which are in general not compatible with rank-dependent models. As it turns out, however, the preferences over

⁸Moreover, our model delivers a unifying perspective on seemingly unrelated phenomena discovered by experimental research, such as the preference for late resolution of uncertainty, hyperbolic and subadditive discounting, the differential discounting of certain and risky prospects, and the order-dependence of prospect valuation. It can also reconcile the magnitude effect in discounting with the magnitude effect in risk taking. Refer to Epper and Fehr-Duda (2015).

the specific future prospects studied in this paper do meet his conditions. Hence, in our model, preferences for resolution timing are an inherent feature of the risk preferences themselves, contrary to the literature based on Kreps and Porteus (1978)'s seminal work in which preferences for resolution timing are modeled with a separate parameter. However, none of these contributions addresses the coexistence of underweighting and overweighting of tail events.

The remainder of the paper is organized as follows: The key assumptions of our model and their implications for general multi-outcome prospects are discussed in Sections 2 and 3. Model predictions are presented in Section 4. Finally, Section 5 concludes. Supplementary materials are available in the appendix where we also show that our results developed for decision under risk are portable to situations in which decision makers do not or cannot know the exact probabilities.

2 Key Assumptions

Our model builds on the basic idea that there is risk attached to any future prospect. The risk inherent in the future, *survival risk* for short, may stem from different sources. At the personal level, it refers to a general feeling of "something may go wrong" due to unexpected contingencies, such as a check getting lost in the mail or involvement in an accident. Another important channel through which survival risk may manifest itself is the institutional environment. Environments where property rights are only weakly protected or institutions of contract enforcement are not reliable, as is the case in many developing countries, are characterized by high survival risk. This risk turns allegedly guaranteed payoffs into risky ones and introduces an additional layer of risk over and above the objective atemporal probability distributions of risky payoffs (henceforth referred to as *base risk*). Consequently, there are two distinct types of risk, time-independent *base risk* and time-dependent *survival risk*. We model the probability of prospect survival by a constant per-period rate s . Thus, the probability that something may go wrong until payoffs materialize at time t amounts to $1 - s^t$.

Since our main concern is the overweighting and underweighting of tail events, we make use of the characteristics of rank-dependent models.⁹ The starting point of our approach is Rank Dependent Utility Theory (RDU).¹⁰ We assume that a decision maker's atemporal risk preferences over prospects that are played out and paid out with negligible time delay can be

⁹For an insightful discussion on the intuition of rank-dependent models see Diecidue and Wakker (2001).

¹⁰RDU is a generalization of expected utility theory and, thus, tacitly also assumes asset integration. While reference dependence, modeled e.g. by Cumulative Prospect Theory (CPT), may be an important additional feature of risk taking behavior, it does not play a role in explaining over- and underweighting of rare extreme events. RDU has several attractive features. First, RDU respects completeness, transitivity, continuity, and first-order stochastic dominance. Moreover, RDU displays first-order attitudes toward risk, i.e. preferences between prospects the consequences of which are sufficiently close to one another do not necessarily tend to risk neutrality. In this sense, experimental evidence favors rank-dependent utility theory over many other non-expected utility approaches that only permit second-order risk aversion (Sugden, 2004). RDU is also able to accommodate correlation aversion (Fehr-Duda and Epper (2012), Appendix A4).

represented by a rank-dependent functional. Consider a prospect $P = (x_1, p_1; \dots; x_m, p_m)$ over (terminal) monetary outcomes $x_1 > x_2 > \dots > x_m \geq 0$ with $\sum p_i = 1$. u maps the utility of monetary amounts x , and w denotes the subjective probability weight attached to p_1 , the probability of the best outcome x_1 . As usual, both u and w are assumed to be monotonically increasing, w to be twice differentiable and to satisfy $w(0) = 0$ and $w(1) = 1$. Decision weights π_i are defined as¹¹

$$\pi_i = \begin{cases} w(p_1) & \text{for } i = 1 \\ w\left(\sum_{k=1}^i p_k\right) - w\left(\sum_{k=1}^{i-1} p_k\right) & \text{for } 1 < i < m \\ 1 - w(1 - p_m) & \text{for } i = m \end{cases} . \quad (1)$$

Thus, the decision weight of x_i is the probability weight attached to the probability of obtaining something at least as good as x_i minus the probability weight attached to the probability of obtaining something strictly better than x_i . Finally, the prospect's value is represented by

$$V(P) = \sum_i^m u(x_i) \pi_i . \quad (2)$$

On average, empirical probability weighting curves are regressive, overweighting small probabilities and underweighting large probabilities (Bruhin, Fehr-Duda, and Epper, 2010), which is also a common pattern in individual data (Gonzalez and Wu, 1999).¹² Common specifications of functional forms for w typically show a combination of concavity over small probabilities and convexity over large probabilities, i.e. an inverse S-shape, which is an additional feature not required for our analysis. A probability weighting function $w(p)$ is regressive if there exists a probability $p^* \in (0, 1)$, such that

$$\begin{aligned} w(p) &> p && \text{for } p < p^* \\ w(p) &= p^* && \text{for } p = p^* \\ w(p) &< p && \text{for } p > p^* \end{aligned} . \quad (3)$$

In the context of rank-dependent models, regressiveness of the probability weighting function generates overweighting of a prospect's tail outcomes and underweighting of its intermediate outcomes, which nicely captures the notion that more extreme outcomes within a given prospect are more salient (see Figure 1 in Section 4.1).

To see why a regressive probability weighting function generates overweighting of the tails, consider Equations 1 and 3. Suppose that both the best and the worst outcomes, x_1 and x_m ,

¹¹Alternatively, decision weights π_i can be expressed in terms of the cumulative distribution function F of the outcomes x_i : $\pi_i = w(1 - F(x_{i+1})) - w(1 - F(x_i))$ for $1 \leq i \leq m$, where $F(x_{m+1}) := 0$.

¹²Aside from regressive shapes, convex weighting curves which globally underweight probabilities comprise another common category of individuals' probability weighting functions (see e.g. van de Kuilen and Wakker (2011)).

materialize with small probabilities (i.e. $m > 2$). The decision weight of the right tail, π_1 , equals $w(p_1)$. As p_1 is small, x_1 is overweighted by w . The decision weight of the left tail, π_m , equals $1 - w(1 - p_m)$. As p_m is small, $1 - p_m$ is large and, hence, underweighted by w . Consequently, x_m is overweighted as well.

Another pervasive feature of risk preferences concerns proneness to Allais-type *common-ratio violations* that constitute one of the most widely replicated experimental regularities in human and animal behavior: Mixing a pair of prospects with common aversive outcomes frequently leads to preference reversals (Allais, 1953; Hagen, 1972; Kahneman and Tversky, 1979; MacCrimmon and Larsson, 1979; Battalio, Kagel, and MacDonald, 1985; Loomes and Sugden, 1987; Kagel, MacDonald, and Battalio, 1990; Nebout and Dubois, 2014; Chark, Chew, and Zhong, 2016).

Inspired by one of Allais (1953)'s famous examples, Kahneman and Tversky (1979) presented subjects with the decision situation summarized in Table 1. In the first decision situation, involv-

Table 1: Allais-Type Common Ratio Pairs

First pair of options:	
\$ 3000 for sure	\succ \$ 4000 with a probability of 80%
Second pair of options:	
\$ 3000 with a probability of 25%	\prec \$ 4000 with a probability of 20%

ing a certain option and a risky one, most people chose the certain option of 3000 dollars. When confronted with the choice between a 25%-chance of receiving 3000 dollars and a 20%-chance of receiving 4000 dollars, the majority opted for the 4000-dollar alternative, however. Multiplying the probabilities of 100% and 80% by a common factor $\lambda \in (0, 1)$, in this example by $\lambda = 1/4$, induced many people to reverse their preferences, a regularity termed *common ratio effect*.

Common-ratio violations are parsimoniously characterized by subproportionality of the probability weighting function w . Formally, subproportionality of w holds for probabilities p and q , if $1 \geq p > q > 0$, and $0 < \lambda < 1$ imply the inequality

$$\frac{w(p)}{w(q)} > \frac{w(\lambda p)}{w(\lambda q)} \quad (4)$$

(Prelec, 1998). Intuitively, subproportionality decreases the decision maker's sensitivity to disappointment for scaled-down probabilities, i.e. outcomes with high ex-ante probabilities of materializing carry higher disappointment potential. In this sense, the loss of certainty hurts more than the scaling down of a probability bounded away from one does. Therefore, subproportionality implies the *certainty effect*, which constitutes the special case of $p = 1$: $w(\lambda q) > w(\lambda)w(q)$ is satisfied for any λ, q such that $0 < \lambda, q < 1$. Many functional specifications proposed in the literature exhibit subproportionality over some probability range under appropriate parameter restrictions

(see Appendix C). Perhaps the most prominent representative of a globally subproportional function with a regressive shape is Prelec (1998)'s flexible two-parameter specification. Throughout the paper, we will use this functional specification to illustrate our results graphically.

3 The Model

Our approach is applicable to an arbitrary number of outcomes provided that survival risk does not change the rank order of the prospects, i.e. "something may go wrong" is encoded as an outcome \underline{x} no better than the prospects' minimum outcome $x_m \geq \underline{x}$. Rearranging terms in Equation 2 yields

$$\begin{aligned} V(P) &= u(x_1)w(p_1) + u(x_2)\left(w(p_1 + p_2) - w(p_1)\right) + \dots + u(x_m)\left(1 - w(1 - p_m)\right) \\ &= \left(u(x_1) - u(x_2)\right)w(p_1) + \dots + \left(u(x_{m-1}) - u(x_m)\right)w(1 - p_m) + u(x_m). \end{aligned} \quad (5)$$

This presentation of V clarifies that x_m is effectively a sure thing whereas obtaining something better than x_m is risky.

If the prospect is not played out and paid out in the present, but at some future time $t > 0$, two additional factors become important. First, we follow the standard approach and model people's willingness to postpone gratification by a constant rate of time preference $\eta \geq 0$, yielding a discount weight of $\rho(t) = \exp(-\eta t)$. This assumption is not crucial for our results - neither a zero rate of time preference, i.e. $\rho = 1$, nor genuinely hyperbolic time preferences affect our conclusions. A prospect to be played out and paid out at $t > 0$ is discounted for time in the standard way:

$$[V(P)]_0 = V(P)\rho(t). \quad (6)$$

Second, and most importantly, survival risk changes the nature of the prospect. Let $0 < s \leq 1$ denote the constant per-period probability of prospect survival, i.e. the probability that the decision maker will actually obtain the promised outcomes by the end of the period.¹³ Then the probability that the allegedly guaranteed payment x_m materializes at the end of period t is perceived to be s^t , and the probabilities of obtaining something better than x_m are scaled down by s^t . Therefore, the objective m -outcome prospect is subjectively perceived as an $(m+1)$ -outcome prospect $\tilde{P} = (x_1, p_1 s^t; x_2, p_2 s^t; \dots; x_m, p_m s^t; \underline{x}, 1 - s^t)$, where \underline{x} captures that "something may go wrong". With the passage of time, the probability of prospect survival gets progressively scaled down.

¹³For similar approaches see Halevy (2008), Walther (2010) and Saito (2015) who study hyperbolic discounting in the context of probability-weighting models.

Setting $u(\underline{x}) = 0$, the subjective present value of the prospect amounts to

$$\begin{aligned}
[V(\tilde{P})]_0 &= \left((u(x_1) - u(x_2))w(p_1s^t) + \dots \right. \\
&\quad \left. \dots + (u(x_{m-1}) - u(x_m))w((1-p_m)s^t) + u(x_m)w(s^t) \right) \rho(t) \\
&= \left((u(x_1) - u(x_2)) \frac{w(p_1s^t)}{w(s^t)} + \dots \right. \\
&\quad \left. \dots + (u(x_{m-1}) - u(x_m)) \frac{w((1-p_m)s^t)}{w(s^t)} + u(x_m) \right) w(s^t) \rho(t).
\end{aligned} \tag{7}$$

Now suppose that the observer assumes that there is no survival risk, i.e. that $s = 1$, while in fact $s < 1$. Consequently, she infers probability weights \tilde{w} and discount weights $\tilde{\rho}$ from observed behavior on the presumption that the decision maker evaluates the objectively given prospect P . However, in the eye of the decision maker the prospect involves an additional layer of risk. If the observer neglects $s < 1$, she infers preference parameters from:

$$[V(\tilde{P})]_0 = \left((u(x_1) - u(x_2))\tilde{w}(p_1) + \dots + (u(x_{m-1}) - u(x_m))\tilde{w}(1-p_m) + u(x_m) \right) \tilde{\rho}(t), \tag{8}$$

interpreting \tilde{w} as true probability weights and $\tilde{\rho}$ as true discount weights, while in fact the weights are distorted by survival risk. Obviously, the measured weights differ from the underlying ones if $s < 1$. By comparing Equation 7 with Equation 8 we can see that the relationship between underlying and observed risk preference parameters is given by

$$\tilde{w}(p) = \tilde{w}(p, t) = \frac{w(ps^t)}{w(s^t)}, \tag{9}$$

as $\tilde{\rho}(t) = w(s^t)\rho(t)$ is interpreted as the discount weight attached to the allegedly certain outcome x_m .¹⁴ Equation 9 defines the central relationship between observed and underlying probability weights. Because $\tilde{w}(p, t) \neq w(p)$ for subproportional preferences, survival risk drives a wedge between atemporal risk preferences and risk taking behavior with respect to delayed prospects. A summary of the model variables is provided in Table 2.

4 Model Predictions

In the following, we present our model predictions rationalizing the over- and underweighting of tail events. As discussed above, both the timing and the process of uncertainty resolution are

¹⁴Time discounting of a certain outcome constitutes the special case of $p = 1$. Concerning the discount weights $\tilde{\rho}(t)$, an equivalent representation was derived by Halevy (2008) for Yaari (1987)'s dual theory with a convex probability weighting function. If \tilde{w} is subproportional, $\tilde{\rho}$ declines hyperbolically (see also Epper, Fehr-Duda, and Bruhin (2011)).

Table 2: Model Variables

	Variable	Description	Characteristics
Prospects	x	monetary payoff	$x \geq 0$
	p	probability of x	$0 \leq p \leq 1$
	s	probability of prospect survival	$0 < s \leq 1$
	$1 - s$	survival risk	
	t	length of time delay	$t \geq 0$
Preferences	$u(x)$	utility function	$u(0) = 0, u' > 0$
	$w(p)$	atemporal probability weight	$w(0) = 0, w(1) = 1, w' > 0$
	η	rate of pure time preference	$\eta \geq 0$, constant
	$\rho(t)$	discount weight	$\rho(t) = \exp(-\eta t)$
Behavior	$\tilde{w}(p, t)$	observed probability weight	$\tilde{w}(p, t) = \frac{w(ps^t)}{w(s^t)}$
	$\tilde{\rho}(t)$	observed discount weight	$\tilde{\rho}(t) = w(s^t)\rho(t)$

crucial features of prospect valuation. We distinguish two cases: First, the prospect is played out and paid out at some time in the future. This situation of one-shot resolution of uncertainty is represented by Theorem 1. Theorem 2 covers the case when uncertainty is resolved sequentially over the course of time.

4.1 One-Shot Resolution of Uncertainty

Turning to the one-shot resolution of base risk and survival risk, we see from Equation 9 that observed probability weights $\tilde{w}(p, t)$ deviate from the underlying atemporal ones $w(p)$ in two respects: First, $w(s^t) < 1$ in the denominator boosts observed weights. Second, $w(ps^t)$ in the numerator distorts observed probability weights. In the following, we suppress delay t in the notation whenever there is no ambiguity about the length of delay. The assumption of subproportional probability weights w generates clear predictions for \tilde{w} :

THEOREM 1:

Given subproportionality of w and $s < 1$:

1. The function \tilde{w} is a proper probability weighting function, i.e. monotonically increasing in p with $\tilde{w}(0) = 0, \tilde{w}(1) = 1$.
2. \tilde{w} is subproportional.

3. \tilde{w} is more elevated than w : $\tilde{w}(p) > w(p)$. Elevation increases with
 - time delay t ,
 - survival risk $1 - s$, and
 - degree of subproportionality.
4. \tilde{w} is less elastic than w .
5. The decision weight of the (objectively) worst possible outcome, x_m , decreases with delay t .

Proof of Theorem 1.

1. Since $\tilde{w}(0) = \frac{w(0)}{w(s^t)} = 0$, $\tilde{w}(1) = \frac{w(s^t)}{w(s^t)} = 1$, and $\tilde{w}' = \frac{w'(ps^t)s^t}{w(s^t)} > 0$ hold, \tilde{w} is a proper probability weighting function.
2. Subproportionality of \tilde{w} follows directly from subproportionality of w as for $p > q$ and $0 < \lambda < 1$:

$$\frac{\tilde{w}(\lambda p)}{\tilde{w}(\lambda q)} = \frac{w(\lambda s^t p)}{w(\lambda s^t q)} < \frac{w(s^t p)}{w(s^t q)} = \frac{\tilde{w}(p)}{\tilde{w}(q)}. \quad (10)$$

3. Since w is subproportional,

$$\tilde{w}(p) = \frac{w(ps^t)}{w(s^t)} > \frac{w(ps)}{w(s)} > \frac{w(p)}{w(1)} = w(p) \quad (11)$$

holds for $s < 1$ and $t > 1$. Therefore, \tilde{w} is more elevated than w . Obviously, elevation gets progressively higher with increasing t and an equivalent effect is produced by decreasing s . Since \tilde{w} increases monotonically in t and $\tilde{w} \leq 1$ for any t , elevation increases at a decreasing rate.

In order to show that a comparatively more subproportional probability weighting function entails a greater increase in observed risk tolerance we examine the relationship between the underlying atemporal probability weights w and observed ones \tilde{w} . Let w_1 and w_2 denote two probability weighting functions, with w_2 exhibiting greater subproportionality.

If $w_1(\lambda)w_1(p) = w_1(\lambda pq)$ holds for a probability $q < 1$, then $w_2(\lambda)w_2(p) < w_2(\lambda pq)$ follows as w_2 is more subproportional than w_1 (Prelec, 1998). Choose $r < 1$ such that $w_2(\lambda)w_2(p) = w_2(\lambda pqr)$. For $\lambda = s^t$, the following relationships hold:

$$\frac{\tilde{w}_1(p)}{w_1(p)} = \frac{w_1(\lambda p)}{w_1(\lambda)w_1(p)} = \frac{w_1(\lambda p)}{w_1(\lambda pq)}. \quad (12)$$

Applying the same logic to w_2 yields

$$\frac{\tilde{w}_2(p)}{w_2(p)} = \frac{w_2(\lambda p)}{w_2(\lambda)w_2(p)} = \frac{w_2(\lambda p)}{w_2(\lambda pqr)} > \frac{w_2(\lambda p)}{w_2(\lambda pq)}. \quad (13)$$

Therefore, the relative wedge $\frac{\tilde{w}_2(p)}{w_2(p)}$ caused by subproportionality is larger than the corresponding one for w_1 .

4. For the elasticity of \tilde{w} , $\varepsilon_{\tilde{w}}(p)$, the following relationship holds:

$$\varepsilon_{\tilde{w}}(p) = \frac{\tilde{w}'(p)p}{\tilde{w}(p)} = \frac{w'(ps^t)ps^t}{w(ps^t)} = \varepsilon_w(ps^t) < \varepsilon_w(p), \quad (14)$$

as the elasticity ε_w increases in its argument iff w is subproportional (Segal, 1987a).

5. As $\tilde{w}(p) > w(p)$ holds for any $0 < p < 1$, $\tilde{\pi}_m = 1 - \tilde{w}(1 - p_m) < 1 - w(1 - p_m) = \pi_m$ results for the decision weight of x_m . As \tilde{w} increases with t , the weight of x_m declines with time delay.

□

That \tilde{w} is more elevated than w constitutes the central implication of our model. Due to subproportionality

$$\frac{w(p_1 s^t)}{w(p_1)} > \frac{w(s^t)}{w(1)}$$

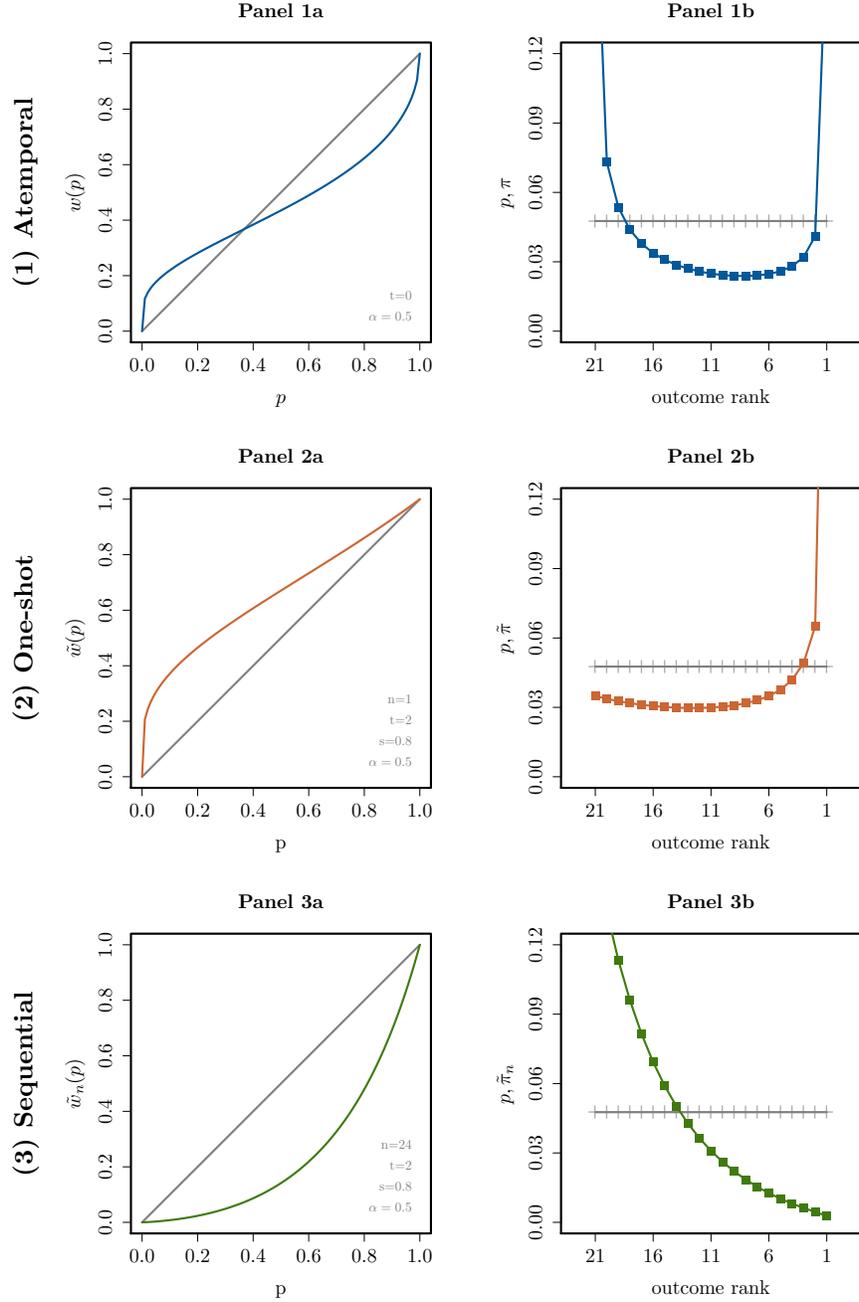
holds, i.e. comparing the delayed case with the atemporal one, the weight of the best possible outcome is devalued less than the weight of the sure component. In other words, x_m suffers more strongly from delay than does x_1 . Thus, the presence of survival risk makes people appear more risk tolerant for delayed prospects than for present ones.¹⁵ The consequences for a regressive w are clear. Figuratively speaking, the decision weight curve rotates counterclockwise: The right tail of the outcome distribution gains more weight, whereas the left tail loses weight with delay. If t is sufficiently large, the left tail may even be underweighted, as illustrated in Figure 1.

The top row of Figure 1 characterizes preferences in the atemporal case. Panel 1a shows a typical specimen of a regressive probability weighting function for delay $t = 0$, underweighting large probabilities and overweighting small probabilities of the best outcome. For illustrative purposes, Panel 1b on the right side depicts the corresponding decision weights for a prospect involving 21 equiprobable outcome levels, with outcome rank 1 denoting the best outcome and outcome rank 21 the worst one. Their objective probabilities are represented on the horizontal gray line. As one can see, a regressive w generates strong overweighting of the extreme outcomes and underweighting of the intermediate ones relative to the objective probability distribution.

The middle row of Figure 1 demonstrates the predictions for one-shot resolution of uncertainty, i.e. when prospects are played out and paid out simultaneously in the future. Future

¹⁵In the domain of simple prospects (x, p) , Baucells and Heukamp (2012) derive a time-dependent probability weighting function $\tilde{w}(p) = w(p \exp(-r_x t))$, which obviously decreases with t . A crucial element of their model is r_x , the probability discount rate that is assumed to decrease with outcome magnitude. It is this assumption that drives their result of risk premia declining with time delay.

Figure 1: Delay Dependence and Process Dependence
 (a) Probability weights (b) Decision weights



For purposes of illustration, the curves are derived from Prelec (1998)'s two-parameter probability weighting function $w(p) = \exp(-\beta(-\ln(p))^\alpha)$, assuming a degree of subproportionality $\alpha = 0.5$ and convexity $\beta = 1$. Survival risk s is set at 0.8 per period. n denotes the number of (equally spaced) stages in the case of sequential evaluation. **Top row (1. atemporal):** The graphs show atemporal probability weights w (Panel 1a) and their associated decision weights π (Panel 1b) for a prospect involving 21 equiprobable outcomes, with outcome rank 1 denoting the best outcome. Their objective probabilities are represented on the horizontal gray line. **Middle row (2. one-shot):** Panel 2a and 2b show \tilde{w} and $\tilde{\pi}$ for a delay of two periods, $t = 2$, when uncertainty resolves in one shot $n = 1$. **Bottom row (3. sequential):** Panel 3a and 3b show \tilde{w} and $\tilde{\pi}$, respectively, for a delay of two periods when uncertainty resolves sequentially in $n = 24$ equally spaced stages, $\tilde{w}(p) = \left(\frac{w((ps^t)^{1/n})}{w((s^t)^{1/n})} \right)^n$.

uncertainty is captured by the parameter $s = 0.8$, i.e. the per-period prospect survival rate is perceived to be 80%. When payoffs are delayed by two periods, $t = 2$, and uncertainty resolves in one shot ($n = 1$) observed probability weights \tilde{w} shift upwards, as shown in Panel 2a. This shift transforms the decision weights as depicted in Panel 2b. Now the worst outcomes are underweighted while the best ones are more strongly overweighted. For longer time delays these effects become more pronounced and may lead to a substantial underweighting of the worst outcomes. Thus, underweighting of adverse extreme events and, hence, underinsuring becomes more likely with longer time horizons. The delay dependence of risk tolerance, therefore, provides a rationale for the underweighting of adverse tail events.

Numerous experimental studies have found that risk tolerance is indeed higher for payoffs materializing in the future than for payoffs materializing in the present (Jones and Johnson, 1973; Shelley, 1994; Ahlbrecht and Weber, 1997; Sagristano, Trope, and Liberman, 2002; Noussair and Wu, 2006; Coble and Lusk, 2010). More specifically, Abdellaoui, Diecidue, and Öncüler (2011) conducted a carefully designed experiment eliciting probability weights for both present and delayed prospects, i.e. in our notation $w(p)$ and $\tilde{w}(p)$. Their results provide persuasive direct support for our approach. They find four distinctive characteristics of delay-dependent prospect valuation. First, the utility for money u does not react to time delay. Second, \tilde{w} is significantly more elevated than w in the aggregate as well as for the majority of the individuals. Third, an additional six-month delay affects elevation less strongly than the first six-month delay. Moreover, \tilde{w} appears to be less strongly curved than w .¹⁶ Another important finding of Abdellaoui, Diecidue, and Öncüler (2011) concerns behavior under timing uncertainty. When their experimental subjects did not know the exact timing of the payoffs, they acted as if the prospects' delays were midway between the present and the longest delay in the experiment, 12 months. This finding suggests that delay dependence is also present in situations when payoff dates, and hence the resolution of uncertainty, are indeterminate.

Aside from delay-dependent risk tolerance, the model produces other interesting effects. For one, \tilde{w} is less elastic than w , implying less sensitivity to anticipated disappointment with respect to delayed prospects. This prediction is in line with Trope and Liberman (2003)'s theory of tem-

¹⁶In their study on ambiguity, Abdellaoui, Baillon, Placido, and Wakker (2011) show estimates of a probability weighting curve derived from choices over prospects delayed by three months. This curve is also much more elevated than typical atemporal estimates are (see for example Bruhin, Fehr-Duda, and Epper (2010)).

poral construal, that posits that temporal distance changes the way people mentally represent those events. The greater the temporal distance, the more likely are events to be represented in terms of a few abstract features. Another insight concerns the role of emotions in the valuation of delayed prospects. Regressiveness and subproportionality can be interpreted as people's reactions to emotions anticipated to occur at the time of uncertainty resolution. Regressiveness maps emotions of elation and disappointment: Elation arises when the best possible outcome materializes in spite of an ex-ante low probability. Disappointment is anticipated to set in when the best possible outcome fails to materialize in spite of an ex-ante high probability (Walther, 2003). Subproportionality can capture the strength of these emotions: The higher the degree of subproportionality, the more pronounced is the departure from linear weighting and, consequently, the reactions to elation and disappointment. This result speaks not only to individual heterogeneity but also to situations that may trigger more or less fear. For example, in times of economic crisis people may react much more strongly to anticipated emotions (Cohn, Engelmann, Fehr, and Marechal, 2015), i.e. they may display a higher degree of subproportionality than in times of economic stability. Thus, in times of crisis, they will react more strongly to imminent risks but more tolerant to risks resolving in the remote future. Finally, the wedge between \tilde{w} and w also increases with the degree of survival risk, implying, somewhat paradoxically, that observed risk tolerance increases with subjective uncertainty.¹⁷

4.2 Sequential Resolution of Uncertainty

So far, we have considered the case of uncertainty resolving in one shot, the domain over which atemporal risk preferences are defined.¹⁸ If uncertainty does not resolve in one shot but rather sequentially over the course of time, future prospects lose their single-stage quality and turn into multi-stage ones. In this case the question arises in which way multi-stage prospects are transformed into single-stage ones. Essentially, there are two different transformation methods, reduction by probability calculus and folding back (Sarin and Wakker, 1994). In the case

¹⁷This finding mirrors Quiggin (2003)'s result of atemporal risk tolerance increasing with background risk.

¹⁸The ramifications of sequential prospect valuation have previously been analyzed for a different class of atemporal risk preferences. Palacios-Huerta (1999)'s contribution focuses on process dependence in the context of Gul (1991)'s model of disappointment aversion. He shows that a disappointment averse decision maker exhibits much larger risk aversion when she evaluates a prospect sequentially rather than in one shot. Dillenberger (2010) provides an axiomatic underpinning for this result and an insightful discussion of the consequences of a preference for one-shot resolution of uncertainty on the value of information. See also Cerreia-Vioglio, Dillenberger, and Ortoleva (2015).

of reduction by probability calculus, the probabilities of reaching the final outcomes are compounded and probability weights are applied only to the resulting compounded probabilities. Folding back means that a multi-stage prospect is evaluated recursively by replacing the n^{th} -stage prospect with its certainty equivalent and inserting the utility of the certainty equivalent into the $(n - 1)^{\text{th}}$ -stage valuation formula and so forth. Thus, decision weights get compounded.

Several authors made a case against reduction as an appropriate mechanism of transforming multi-stage prospects into single-stage ones (e.g. Segal (1990); Dekel, Safra, and Segal (1991); Grant, Kajii, and Polak (1998)). Segal (1990) argues that even if the decision maker accepts the basic laws of probability theory she may have a preference over the number of lotteries she participates in, which invalidates reduction by probability calculus.

However, non-EU preferences raise the issue of dynamic consistency. Dynamic consistency requires that choices or plans made at different times conform with one another (Sugden, 2004). As Loomes and Sugden (1986) explain, any theory that accommodates the common-ratio effect must dispense either with dynamic consistency or with reduction by the probability calculus. Therefore, if the decision maker cares only about the total probabilities of the final outcomes she will be dynamically inconsistent unless she precommits herself to stick to her original plans.¹⁹ Folding back, on the other hand, ensures dynamic consistency but, as Theorem 2 will show, has substantial consequences for revealed risk taking behavior (see also Sarin and Wakker (1992)). In the following, we set $\rho = 1$ for ease of exposition.

Let us first consider a two-outcome prospect $P = (x_1, p; x_2)$ resolving in two stages, $n = 2$, such that uncertainty is partially resolved at some future time t_1 and fully resolved at the payment date $t > t_1$, as depicted in Figure 2. Applying folding back, the resulting two-stage prospect is evaluated as

$$\begin{aligned}
[V_2(\tilde{P})]_0 &= \left(u(x_1) - u(x_2) \right) w \left(p^{\frac{t_1}{t}} s^{t_1} \right) w \left(p^{\frac{t-t_1}{t}} s^{t-t_1} \right) + u(x_2) w \left(s^{t_1} \right) w \left(s^{t-t_1} \right) \\
&= \left(\left(u(x_1) - u(x_2) \right) \frac{w \left(p^{\frac{t_1}{t}} s^{t_1} \right) w \left(p^{\frac{t-t_1}{t}} s^{t-t_1} \right)}{w \left(s^{t_1} \right) w \left(s^{t-t_1} \right)} + u(x_2) \right) w \left(s^{t_1} \right) w \left(s^{t-t_1} \right) \quad (15) \\
&= \left(\left(u(x_1) - u(x_2) \right) \tilde{w}_2(p) + u(x_2) \right) \tilde{\rho}_2(t),
\end{aligned}$$

¹⁹A time-inconsistent decision maker will become progressively less risk tolerant as the payment date draws nearer.

which yields the relationship

$$\tilde{w}_2(p) = \frac{w\left(p^{\frac{t_1}{t}} s^{t_1}\right) w\left(p^{\frac{t-t_1}{t}} s^{t-t_1}\right)}{w\left(s^{t_1}\right) w\left(s^{t-t_1}\right)} \quad (16)$$

as $\tilde{\rho}_2(t) = w\left(s^{t_1}\right) w\left(s^{t-t_1}\right)$ is interpreted as the discount weight attached to the allegedly certain outcome x_2 . Subproportionality ensures that

$$\tilde{w}_2(p) = \frac{w\left(p^{\frac{t_1}{t}} s^{t_1}\right) w\left(p^{\frac{t-t_1}{t}} s^{t-t_1}\right)}{w\left(s^{t_1}\right) w\left(s^{t-t_1}\right)} < \frac{w\left(p s^t\right)}{w\left(s^t\right)} = \tilde{w}(p), \quad (17)$$

the main result of Theorem 2. Now suppose that the interval $[0, t]$ is partitioned into n subintervals with lengths $\tau_i, i \in \{1, \dots, n\}$, such that $\sum_{i=1}^n \tau_i = t$. In this case, it is straightforward to show for any number of outcomes $m \geq 1$ that the observed probability weights are given by

$$\tilde{w}_n(p, t) = \frac{\prod_{i=1}^n w\left(p^{\frac{\tau_i}{t}} s^{\tau_i}\right)}{\prod_{i=1}^n w\left(s^{\tau_i}\right)} = \prod_{i=1}^n \tilde{w}\left(p^{\frac{\tau_i}{t}}, \tau_i\right). \quad (18)$$

THEOREM 2:

Given subproportionality of $w, s \leq 1$ and folding back:

1. Risk tolerance is higher for one-shot resolution of uncertainty than for sequential resolution of uncertainty: $\tilde{w}(p, t) > \tilde{w}_n(p, t)$.
2. For a given number of evaluation stages n , prospect valuation is lowest for equally spaced subintervals $\tau_i = \frac{t}{n} = \tau$.²⁰
3. For equally spaced subintervals, prospect valuation declines with the number of evaluation stages: $[\tilde{V}_n]_0 < [\tilde{V}_{n-1}]_0$.

Proof of Theorem 2.

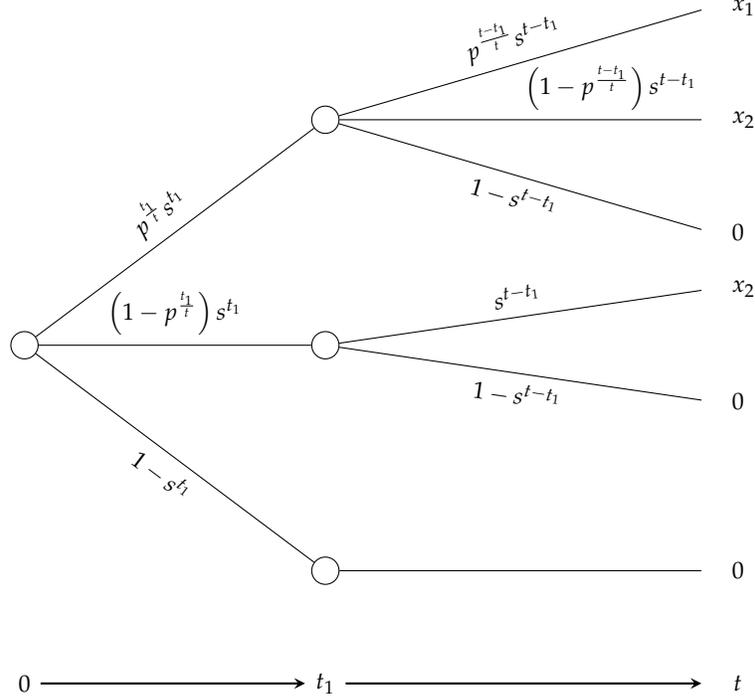
1. Consider Equation 18:

$$\tilde{w}_n(p, t) = \prod_{i=1}^n \tilde{w}\left(p^{\frac{\tau_i}{t}}, \tau_i\right).$$

$$\text{Note that } \tilde{w}\left(p^{\frac{\tau_i}{t}}, \tau_i\right) = \frac{w\left(p^{\frac{\tau_i}{t}} s^{\tau_i}\right)}{w\left(s^{\tau_i}\right)} < \frac{w\left(p^{\frac{\tau_i}{t}} s^{\tau_i} s^{t-\tau_i}\right)}{w\left(s^{\tau_i} s^{t-\tau_i}\right)} = \frac{w\left(p^{\frac{\tau_i}{t}} s^t\right)}{w\left(s^t\right)} = \tilde{w}\left(p^{\frac{\tau_i}{t}}, t\right).$$

²⁰For \tilde{w}_n to be smallest for equally spaced partitions, an additional condition is required: the elasticity of w has to be convex.

Figure 2: Sequential Resolution of Uncertainty



According to Theorem 1, $\tilde{w}(p, t)$ is subproportional for a fixed length of delay t and, therefore,

$$\tilde{w}_n(p, t) < \prod_{i=1}^n \tilde{w}\left(p^{\frac{\tau_i}{t}}, t\right) < \tilde{w}\left(\prod_{i=1}^n p^{\frac{\tau_i}{t}}, t\right) = \tilde{w}(p, t). \quad (19)$$

- Without loss of generality, we reorder the sequence of subintervals such that $\tau_1 \leq \tau_2 \leq \dots \leq \tau_n$. For some i , $\tau_{i-1} < \tau_i$ holds because otherwise the partition would be equally spaced right away. In this case, there exists $\varepsilon > 0$ such that $\tau_{i-1} + \varepsilon < \tau_i - \varepsilon$ is still satisfied. Due to subproportionality, the following relationships hold for $0 < q < 1$:

$$\frac{w(q^{\tau_{i-1}})}{w(q^{\tau_i - \varepsilon})} > \frac{w(q^{\tau_{i-1} + \varepsilon})}{w(q^{\tau_i - \varepsilon})} = \frac{w(q^{\tau_{i-1} + \varepsilon})}{w(q^{\tau_i})}, \quad (20)$$

implying $w(q^{\tau_{i-1}})w(q^{\tau_i}) > w(q^{\tau_i - \varepsilon})w(q^{\tau_{i-1} + \varepsilon})$, in particular for probabilities $q = p^{1/t}s$ and $q = s$. Therefore, compounding probability weights and decision weights over a more evenly spaced partition generates a smaller prospect value.

- Consider two equally spaced partitions of $[0, t]$: $(\tau_i = \frac{t}{n} =: \tau)_{i=1, \dots, n}$ and $(\delta_i = \frac{t}{n-1} =: \delta)_{i=1, \dots, n-1}$. Our claim is that for $0 < p \leq 1$,

$$\prod_{i=1}^n w(p^{\frac{\tau}{i}} s^{\tau}) < \prod_{i=1}^{n-1} w(p^{\frac{\delta}{i}} s^{\delta}). \quad (21)$$

Setting $q = (p^{\frac{1}{i}} s)^{\frac{t}{n(n-1)}}$, we examine whether

$$\left(w(q^{n-1})\right)^n < \left(w(q^n)\right)^{n-1}. \quad (22)$$

Proceeding by complete induction:

- $n = 2$: Subproportionality implies $(w(q))^2 < w(q^2)$.
- $n = 3$: Subproportionality implies $w(q^3) > \frac{(w(q^2))^2}{w(q)}$. Thus,

$$\begin{aligned} (w(q^3))^2 &> \frac{(w(q^2))^2}{w(q)} \frac{(w(q^2))^2}{w(q)} > \frac{(w(q^2))^3 w(q^2)}{(w(q))^2} \\ &> \frac{(w(q^2))^3 (w(q))^2}{(w(q))^2} = (w(q^2))^3 \end{aligned} \quad (23)$$

- $n \rightarrow n + 1$: Suppose that $(w(q^{n-1}))^n < (w(q^n))^{n-1}$ holds. Subproportionality implies $\frac{w(q^{n-1})}{w(q^n)} > \frac{w(q^n)}{w(q^{n+1})}$. Hence,

$$\begin{aligned} (w(q^{n+1}))^n &> \left(\frac{w(q^n)w(q^n)}{w(q^{n-1})}\right)^n = \frac{(w(q^n))^{n+1} (w(q^n))^{n-1}}{(w(q^{n-1}))^n} \\ &> \frac{(w(q^n))^{n+1} (w(q^{n-1}))^n}{(w(q^{n-1}))^n} = (w(q^n))^{n+1} \end{aligned} \quad (24)$$

□

Theorem 2 shows that a decision maker with subproportional preferences prefers uncertainty to be resolved in one shot at the payment date t rather than sequentially over the course of time.²¹

²¹A special case is the valuation of allegedly certain future payoffs, which constitute simple prospects in our framework. A myopic decision maker, applying folding back, will exhibit a discount weight of $w(s^{t_1}) w(s^{t-t_1}) < w(s^t)$, an incident of *subadditive discounting*, which has found experimental support (Read, 2001; Read and Roelofsma, 2003; Ebert and Prelec, 2007; Epper, Fehr-Duda, and Bruhin, 2009; Dohmen, Falk, Huffman, and Sunde, 2012).

Note that this result does not hold generally under subproportionality in RDU but only applies to the class of prospects studied here, i.e. prospects that are devalued by survival risk without effects on the rank order of the outcomes (see Dillenberger (2010)'s necessary and sufficient criterion for preferences for one-shot resolution and our discussion in Appendix B).

Preference for one-shot resolution of uncertainty is embodied in the characteristics of atemporal risk preferences and, therefore, all the insights of Segal (1990), who analyzes two-stage prospects in an atemporal setting, still apply. However, risk tolerance is additionally influenced by its delay dependence. Consider a prospect with a long time horizon t . If its total uncertainty is resolved in one single stage, all the decision weights attain their maximum values. If uncertainty resolves sequentially, both probability and discount weights are smaller than in the one-shot case. The effect gets more pronounced the finer is the partition of delay t into subintervals. Therefore, anticipating to watch uncertainty resolve over time considerably dampens the effect of long time horizons on observed risk tolerance, because the decision maker is frequently exposed to the possibility of a disappointing outcome.

Our model predicts that equally spaced partitions of the time interval will be valued particularly unfavorably. Partitions of equal length correspond to the least degenerate multi-stage prospect and can be interpreted as the comparatively most ambiguous situation, which is strongly disliked by people with subproportional preferences. Because of this characteristic, Segal (1987b) proposes to model ambiguity aversion by subproportional risk preferences over two-stage lotteries.²²

The consequences of sequential valuation for the tails of the outcome distribution are straight forward if w is regressive. The weight of the right tail $\tilde{\pi}_n(x_1) = \tilde{w}_n(p_1) < \tilde{w}(p_1)$. Therefore, overweighting of x_1 declines relative to the one-shot situation. The weight of the left tail increases as $\tilde{\pi}_n(x_m) = 1 - \tilde{w}_n(1 - p_m) > 1 - \tilde{w}(1 - p_m)$ holds. Depending on the number of subperiods over which probability weights are compounded, this increase may lead to a considerable overweighting of the worst outcomes and, consequently, to pronounced risk aversion.

The bottom row of Figure 1 demonstrates the effect of sequential valuation on probability weights and decision weights for a delay of $t = 2$. If a prospect is evaluated in 24 equally spaced

²²A recent paper by Dillenberger and Segal (2014) shows that such an approach has another attractive implication: It is able to solve Machina (2009, 2014)'s paradoxes which involve a number of situations where standard models of ambiguity aversion are unable to capture plausible features of ambiguity attitudes (Baillon, l'Haridon, and Placido, 2011).

time intervals, $n = 24$, the probability weighting curve takes on a convex form, which implies strong risk aversion. The associated decision weights for our reference prospect involving 21 equiprobable outcomes are depicted in Panel 3b. The decision weight curve now rotates clockwise: The worst outcomes are strongly overweighted while the best outcomes are considerably underweighted. Sequential valuation, therefore, has a dramatic effect on the overweighting of adverse tail events. This effect may be called *myopic probability weighting* in the style of myopic loss aversion (Benartzi and Thaler, 1995) which has similar consequences on risk taking behavior when short-sighted investors are frequently exposed to the possibility of incurring losses.

5 Discussion

Most economically important decisions, may they concern health, wealth, love or education involve a significant interval between the time that the decision is made and the time that all uncertainty is completely resolved. Our contribution provides a novel view on perplexing real-world behaviors. We show that if people view the future as inherently uncertain and are susceptible to probability weighting, their risk tolerance varies greatly depending on the length of delay and their perception of uncertainty resolution. When the passage of time does not play a significant role, a typical decision maker overweights both tails of an outcome distribution. This feature of risk preferences explains people's skewness preferences, favoring positively skewed distributions and disfavoring negatively skewed ones (Lovallo and Kahneman, 2000; Ebert and Wiesen, 2011; Barberis, 2013b; Ebert, 2015). If uncertainty resolves in the future, however, adverse tail events receive progressively less weight and, for long time horizons, may even be substantially underweighted, thereby greatly reducing people's willingness to buy insurance.

Delay- and process-dependent risk tolerance not only affects individuals' welfare but also society at large. People's reluctance to take out insurance for floods and earthquakes, for example, poses serious problems when disaster actually strikes. It is practically impossible for the public authorities to deny assistance once there are identified victims and their stories are publicized in the news (Viscusi, 2010). In the context of climate policy, it takes decades or even centuries until the stock of pollutants will be sufficiently reduced to see any gaugeable effect of society's abatement endeavors. If there is both great uncertainty about the effectiveness of abatement policies and lack of feedback, the risk tolerance of a large percentage of the population may be extremely

high and, therefore, it is likely that they are opposed to supporting abatement measures. It remains to be seen whether endeavors to combat global warming will be met with more support once its effects become more visible.

Stock market investors' time horizons may also be long-term in principle but, contrary to natural disasters, information on portfolio performance is easily accessible and, due to its omnipresence in the news, hard to ignore. Thus, uncertainty resolves practically in real time, which substantially counteracts the otherwise risk-tolerance increasing effect of long investment horizons. Recently, the term structure of market risk premia has attracted considerable attention (Andries, Eisenbach, and Schmalz, 2015; Eisenbach and Schmalz, 2016). The empirical evidence points to a downward sloping curve, i.e. assets with short maturities seem to earn much higher risk premia than assets with long maturities (van Binsbergen, Brandt, and Koijen, 2012), which contradicts the predictions of standard asset-pricing models. Consequently, new models of asset pricing work with the assumption of horizon-dependent risk tolerance (Khapko, 2015). Our model provides a rationale for both, high risk premia, because of the gradual resolution of uncertainty, and risk premia declining with maturity, because of the delay-dependence of risk tolerance.

Referring to experimental evidence in atemporal settings, Hertwig, Barron, Weber, and Erev (2004) suggest an alternative explanation for the underweighting of tail events. They argue that overweighting occurs in situations when risks are described in abstract terms. However, when people decide on the basis of their own experience by sampling the distributions, they tend to underweight tail events. This claim has triggered a heated debate on the so-called description-experience gap (Barberis, 2013a; de Palma, Abdellaoui, Attanasi, Ben-Akiva, Erev, Fehr-Duda, Fok, Fox, Hertwig, Picard, Wakker, Walker, and Weber, 2014). Recently, Abdellaoui, L'Haridon, and Paraschiv (2011) showed that, contrary to the case of fully described risks, having to find out themselves about outcomes and probabilities by experience sampling makes people considerably more pessimistic, which manifests itself in a less elevated probability weighting curve. In other words, ambiguity about distributions shifts the probability weighting curve downwards, which may explain the underweighting of rare extreme events observed in experiments. Many empirical facts in finance, insurance and gambling are consistent with the overweighting of tail events, however. According to Hertwig, Barron, Weber, and Erev (2004)'s claim all these phenomena

would have to be based on described risks. In our view, it seems implausible that in many real-world situations people's decisions are based solely on abstract descriptions rather than on their own or somebody else's experience. Turning back to our example in the introduction: Why should choices over a regular life insurance policy be driven by experience and choices over a flight insurance policy by description?

Models of probability weighting have proven to be quite successful in organizing the results of countless experiments. Recently, it has been recognized that they are useful for explaining field data as well. Here we show that extending the realm of probability weighting from timeless decisions to intertemporal ones helps rationalize the coexistence of over- and underweighting of tail events, a puzzle unsolved so far. Whether the mechanism we suggest is actually driving behavior needs to be assessed by future work. The model presented in this paper provides a host of novel testable predictions which, we hope, will encourage researchers to conduct experiments to gauge the extent of its applicability.

Appendix A The Case of Ambiguity

In real-world settings probabilities are rarely known to the decision maker. With the exception of some games of chance, such as tossing a coin or playing roulette, the decision maker has to assess the likelihoods of ambiguous events. Our model is cast in terms of objectively given probabilities, however. Thus, the question arises whether our results are portable to the domain of ambiguity.

In this domain, the following framework is usually applied: S is a set of exhaustive and mutually exclusive states of nature. One of these states $s \in S$ will obtain, but the decision maker is unsure which one it will be. Subsets of S are called events and denoted by A . Prospects, often termed *acts*, are described as $P = (x_1, A_1; \dots; x_m, A_m)$, which yield the monetary outcome if the event A_i contains the true state of nature. Outcomes are rank ordered $x_i, x_1 > x_2 > \dots > x_m$ and (A_1, A_2, \dots, A_m) is a partition of the state space. To accommodate ambiguity, RDU is generalized to Choquet Expected Utility Theory (Schmeidler, 1989), which features a weighting function $W(A)$. W is a capacity satisfying $W(\emptyset) = 0, W(S) = 1$, and monotonicity with respect to set inclusion, i.e. $A \subset B \implies W(A) \leq W(B)$. Decision weights π_i are constructed analogously to the case of risk:

$$\pi_i = \begin{cases} W(A_1) & \text{for } i = 1, \\ W\left(\bigcup_{k=1}^i A_k\right) - W\left(\bigcup_{k=1}^{i-1} A_k\right) & \text{for } 1 < i \leq m. \end{cases} \quad (25)$$

As before, the prospect's value is represented by

$$V(P) = \sum_i^m u(x_i)\pi_i. \quad (26)$$

There is a large literature in the psychology of judgment which suggests that, generally, people tend to overweight the likelihood of rare events and underweight the likelihood of probable events. A prominent example are the frequency estimates for causes of death reported in Tversky and Koehler (1994). The same pattern of behavior has been found in experimental research on decisions under ambiguity (Tversky and Wakker, 1995; Gonzalez and Wu, 1999; Kilka and Weber, 2001; Abdellaoui, Vossman, and Weber, 2005). In the literature, this pattern of overweighting and underweighting is discussed under the heading of *subadditivity*, a consequence of diminishing sensitivity towards probabilities when moving away from certainty and impossi-

bility (Einhorn and Hogarth, 1985; Fox and See, 1993; Wakker, 2004).²³ Formally, subadditivity comprises two conditions, *lower subadditivity* \underline{SA} and *upper subadditivity* \overline{SA} :

$$\underline{SA} : W(A) \geq W(A \cup B) - W(B), \quad (27)$$

$$\overline{SA} : 1 - W(S - A) \geq W(A \cup B) - W(B), \quad (28)$$

provided that $A \cap B = \emptyset$ and $W(A \cup B)$ and $W(B)$ are bounded away from 1 and 0, respectively.²⁴

Experimental studies suggest that subadditivity is more pronounced under ambiguity than under risk, which induced Tversky and Fox (1995) to suggest a two-stage model, formalized in Wakker (2004): Consider an ambiguous prospect (x, A) that pays x in the event that A occurs and zero otherwise. Furthermore, assume that its value can be represented by $V((x, A)) = u(x)W(A)$. Elicit the matching probability $\hat{p}(A)$ such that the decision maker is indifferent between the risky prospect (x, \hat{p}) and the ambiguous prospect $(x; A)$. Then $W(A)$ can be decomposed as

$$W(A) = w(\hat{p}(A)), \quad (29)$$

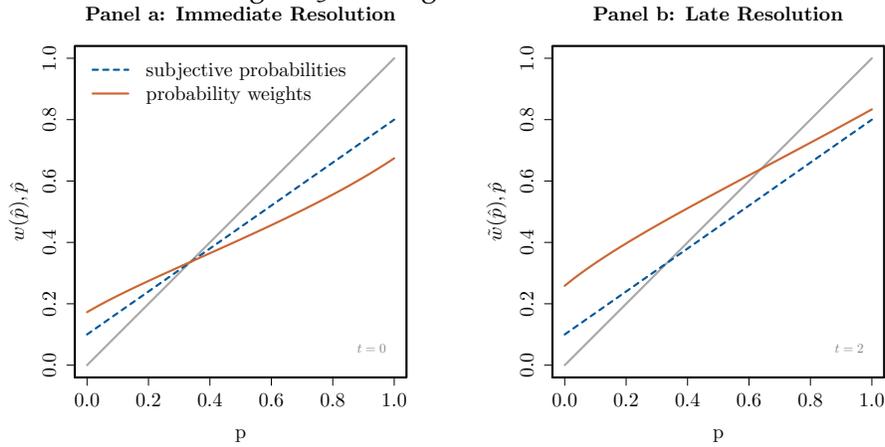
where w is the probability weighting function for risk. This decomposition has been used in a number of experimental studies (Abdellaoui, Vossman, and Weber, 2005; Baillon, 2008; Baillon, Huang, Selim, and Wakker, 2016). The probability weighting function w for decisions under ambiguity has been found to differ from the pure risk case in that it is more strongly subadditive (Abdellaoui, Baillon, Placido, and Wakker, 2011), with the degree of departure from the risk case depending on the *source* of uncertainty, i.e. the concrete decision context (i.e. whether ambiguity concerns the composition of Ellsberg urns, the temperature in a specific city the following day, the movement of a specific stock index, etc.).

The crucial link to our analysis is that subadditivity is implied by strong regressiveness of the probability weighting function (for a proof see Prelec (1998), footnote 10). Therefore, all our predictions also apply to the case of ambiguity. We use these insights to develop a graphical representation of such a two-stage model which serves as basis for illustrating the effects of delay on behavior under ambiguity.

²³Subadditivity also drives the famous Allais common consequence effect (Wu and Gonzalez, 1998).

²⁴An example involving decision weights akin to our approach can be found in Chateauneuf, Eichberger, and Grant (2007).

Figure 3: Ambiguous Probabilities



Panel a: The dashed curve corresponds to subjective probabilities \hat{p} assumed to follow the regularity $\hat{p} = 0.1 + 0.7p$, where p denotes empirical frequency. $w(p)$ is Prelec’s functional specification with $\alpha = 0.64$ and $\beta = 1.03$ applied to \hat{p} (solid curve). **Panel b:** The solid curve depicts $\tilde{w}(\hat{p})$ constructed according to Equation 9 with $s = 0.8$ and $t = 2$.

The curves in Figure 3 are constructed in the following way: In these graphs, probabilities p are in principle objectively given but the decision maker does not know them with precision, for example empirically observed frequencies of specific events. The decision maker judges the likelihoods \hat{p} of these events (or reports choice-based probabilities). These probabilities are represented by the dashed lines in Figure 3, which mimic typical findings on the relationship between subjective judgments \hat{p} and observed frequencies p (e.g. Fox and Tversky (1998), Figure 5). Applying w with parameters found in the literature (Abdellaoui, Baillon, Placido, and Wakker, 2011) to \hat{p} renders the probability weighting curve $w(p)$ as a function of observed frequencies (solid curves in the figure). Panel a shows atemporal preferences, whereas Panel b depicts the case of delaying payoffs by 2 periods. As one can see, the resulting probability weighting curves for ambiguity are much less strongly curved than in the risky situation displayed in Figure 1, but qualitatively the same implications for behavior over delayed prospects arise.

Appendix B A Note on Sequential Evaluation

In his Proposition 1, Dillenberger (2010) shows that, under recursivity, negative certainty independence (NCI) and a weak preference for one-shot resolution of uncertainty (PORU) are

equivalent. The NCI axiom requires the following to hold: If a prospect $P = (x_1, r; x_2)$ is weakly preferred to a degenerate prospect $D = (y, 1)$, then mixing both with any other prospect does not result in the mixture of the degenerate prospect D being preferred to the mixture of P . This axiom is weaker than the standard independence axiom and does not put any restrictions on the reverse preference relation when a degenerate prospect is originally preferred to a nondegenerate one. The latter case characterizes the typical Allais certainty effect. NCI allows for Allais-type preference reversals but does not imply them. Dillenberger's Proposition 3 demonstrates that NCI is generally incompatible with rank-dependent utility unless the probability weighting function is linear, i.e. unless RDU collapses to EUT. An intuitive explanation for Dillenberger's Proposition 3 is that under RDU prospect valuation is sensitive to the rank order of the outcomes and, therefore, mixtures with other prospects may affect the original rank order of outcomes in P (and D). How does Dillenberger's result relate to our claim that subproportional probability weights conjointly with recursivity imply a strong preference for one-shot resolution of uncertainty?

The crucial insight is that for the class of prospects studied in this paper changes in rank order do not occur and, hence, NCI is satisfied. To see this, assume that the prospect $(x_1, p; x_2)$, $x_1 > x_2 \geq 0$, gets resolved in two stages $\left((x_1, r; x_2), q; (x_2, 1) \right)$ such that $p = qr$. In the atemporal case, when there is no additional survival risk, the two-stage prospect continues to be a strictly two-outcome one and the only relevant mixtures are those involving x_2 . Suppose that $P = (x_1, r; x_2) \succeq (y, 1) = D$, with $x_1 > y > x_2$ and consider the following mixtures with $(x_2, 1 - \lambda)$ for some $\lambda \in (0, 1)$: $P' = (x_1, \lambda r; x_2)$ and $D' = (y, \lambda; x_2)$. The following relationships hold:

$$\begin{aligned}
P \succeq D &\Rightarrow V(P) = \left(u(x_1) - u(x_2) \right) w(r) + u(x_2) \geq u(y) = V(D) \\
V(D') &= u(y)w(\lambda) + u(x_2)(1 - w(\lambda)) \\
&\leq \left(\left(u(x_1) - u(x_2) \right) w(r) + u(x_2) \right) w(\lambda) + u(x_2)(1 - w(\lambda)) \\
&= \left(u(x_2) - u(x_1) \right) w(r)w(\lambda) + u(x_2) \\
&< \left(u(x_2) - u(x_1) \right) w(\lambda r) + u(x_2) \\
&= V(P')
\end{aligned} \tag{30}$$

because $w(r)w(\lambda) < w(\lambda r)$ for any $\lambda \in (0, 1)$ (and hence also for $\lambda = q$) due to subproportionality of w . Consequently, for mixtures with the smaller outcome x_2 , NCI, and therefore also PORU,

is *strongly* satisfied. If the mixing prospect may be any arbitrary prospect, in other words if surprises are possible in the course of uncertainty resolution, this result does not hold generally. The only surprise that is still admissible is the occurrence of an outcome worse than x_2 , say z . Define $P'' = (x_1, \lambda r; x_2, \lambda(1-r); z)$ and $D'' = (y, \lambda; z)$.

$$\begin{aligned}
V(D'') &= u(y)w(\lambda) + u(z)(1 - w(\lambda)) \\
&\leq \left((u(x_1) - u(x_2))w(r) + u(x_2) \right) w(\lambda) + u(z)(1 - w(\lambda)) \\
&= (u(x_1) - u(x_2))w(r)w(\lambda) + (u(x_2) - u(z))w(\lambda) + u(z) \\
&< (u(x_1) - u(x_2))w(\lambda r) + (u(x_2) - u(z))w(\lambda) + u(z) \\
&= V(P'')
\end{aligned} \tag{31}$$

For $u(z) = 0$, this case is exactly the situation studied in this paper when survival risk comes into play.

Appendix C Subproportionality

In this section we review a number of probability weighting functions that are either globally or locally subproportional. We limit our attention to functional forms with at most two parameters. Recall that subproportionality is equivalent to increasing elasticity. Consequently, if the elasticity is U-shaped, the function is superproportional over the range of small probabilities and subproportional over large probabilities. These functions capture the certainty effect but not necessarily general common-ratio violations. Many specifications used in the literature exhibit such a characteristic. Some experimenters found reverse common-ratio violations which require superproportionality over the relevant probability range (see e.g. Blavatskyy (2010)). Ultimately, it is an empirical issue whether locally or globally subproportional functions fit better.

Polynomials are linear in the parameters and, thus, generally less flexible than specifications that are nonlinear in the parameters. Note that second-order polynomials demarcate the intersection of the class of quadratic utility and RDU (see also the discussion in Masatlioglu and Raymond (2016)).

Gul (1991)'s theory of disappointment aversion, for example, implies a strictly convex subproportional function in the context of RDU for two-outcome prospects. Another interesting

specimen is the probability weighting function discussed in Delquié and Cillo (2006). In the context of RDU, their model of disappointment aversion generates a subproportional second-order polynomial that is equivalent to the one implied by Köszegi and Rabin (2007)'s choice-acclimating personal equilibrium, which provides an endogenous reference point (Masatlioglu and Raymond, 2016). The same polynomial also emerges in Safra and Segal (1998)'s approach to constant risk aversion. This concept captures the idea that a decision maker commits to a choice long before uncertainty is resolved, and is, therefore, particularly plausible in the context of our model. Bordalo, Gennaioli, and Shleifer (2012) derive (discontinuous) context-dependent probability distortions from their salience theory. While their concave segment is superproportional, the convex segment is equivalent to a subproportional probability weighting function of the Rachlin, Raineri, and Cross (1991) variety. The psychological mechanisms underlying probability weighting, therefore, often imply subproportionality.

Table 3: Probability Weighting Functions

Probability weighting function $w(p)$	Parameter range	Elasticity	Shape	Reference
p^α	$\alpha > 1$	constant	convex	Luce, Mellers, and Chang (1993)
$\exp(-\beta(-\ln(p))^\alpha)$	$0 < \alpha < 1, \beta > 0$	increasing, concave/convex	inverse S	Prelec (1998)
	$\alpha = 1, \beta > 1$	constant	convex	Prelec (1998) ¹
$\frac{p^\alpha}{p^\alpha + (1-p)^\alpha}^{1/\alpha}$	$0.279 < \alpha < 1$	U-shaped	inverse S	Tversky and Kahneman (1992)
$\frac{\beta p^\alpha}{\beta p^\alpha + (1-p)^\alpha}$	$0 < \alpha < 1, \beta > 0$	U-shaped	inverse S	Goldstein and Einhorn (1987)
	$0 < \alpha < 1, \beta = 1$	U-shaped	inverse S	Karmarkar (1979)
	$\alpha = 1, \beta < 1$	increasing, convex	convex	Rachlin, Raineri, and Cross (1991)
		see text	see text	Bordalo, Gennaioli, and Shleifer (2012) ²
$\frac{p + \alpha p(1-p)}{1 + (\alpha + \beta)p(1-p)}$	$\alpha > 0, \beta > 0$	U-shaped	inverse S	Walther (2003)
$\begin{cases} \beta^{1-\alpha} p^\alpha & \text{if (i) } 0 \leq p \leq \beta \\ 1 - (1 - \beta)^{1-\alpha} (1 - p)^\alpha & \text{if (ii) } \beta < p \leq 1 \end{cases}$	$0 < \alpha, \beta < 1$	(i) constant, (ii) increasing	inverse S	Abdellaoui, l'Haridon, and Zank (2010) ³
$\frac{1}{1 + \alpha(1-p)}$	$\alpha > 0$	increasing, convex	convex	Gul (1991)
$p - \alpha p + \alpha p^2$	$0 < \alpha < 1$	increasing, concave	convex	Masatlioglu and Raymond (2016); Delquié and Cillo (2006); Safra and Segal (1998) ⁴
$p + \frac{3-3\beta}{\alpha^2-\alpha+1}(\alpha p - (\alpha+1)p^2 + p^3)$	$0 < \alpha, \beta < 1$	U-shaped	inverse S	Rieger and Wang (2006)
$p - \alpha p(1-p) + \beta p(1-p)(1-2p)$	α depends on β	variety	variety	Blavatsky (2014) ⁵
$\begin{cases} 0 & \text{for } p = 0 \\ \beta + \alpha p & \text{for } 0 < p < 1 \\ 1 & \text{for } p = 1 \end{cases}$	$0 \leq \beta < 1, 0 < \alpha \leq 1 - \beta$	increasing	inverse S	Bell (1985); Cohen (1992); Chateauneuf, Eichberger, and Grant (2007)

(1) Equivalent to power specification $w(p) = p^\beta$.

(2) The weighting function consists of a concave and a convex segment with a jump discontinuity in between.

(3) For $\alpha > 1, \beta = 1$ constant elasticity, convex; for $\alpha < 1, \beta = 0$ increasing elasticity, convex.

(4) Special case of Blavatsky (2014) with $\beta = 0$.

(5) Specific parameter constellations with $\beta > 0$ generate inverse S with U-shaped elasticity.

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