

Income Expectations, Limited Liquidity, and Anomalies in Intertemporal Choice

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Abstract

Elicited discount rates typically lie far beyond market interest rates, tend to decline in time horizon and in outcome magnitude, and appear to be larger for gains than for losses. It has been concluded that these facts violate the economic standard model of intertemporal choice, exponential discounted utility, and that they can only be accommodated by introducing preferences which fundamentally depart from standard ones. In contrast to this conclusion, we demonstrate that all the apparent anomalies can be rationalized within the bounds of the standard model if access to liquidity is limited and income expectations are positive. Intuitively, liquidity-constrained agents who dislike fluctuations in their consumption path, but expect their income to rise in a not so distant future prefer to allocate newly available cash inflows at dates earlier than their pure rate of time preference suggests. This mechanism builds on assumptions which are plausible to hold for typical participants in studies reporting the anomalies, and it is operative when stakes are comparatively small. Furthermore, recent empirical evidence renders direct support for our rationalization. We obtain a number of novel testable predictions, and show that our explanation can resolve a series of puzzling findings which have remained largely unexplained so far, such as (i) the observation that a substantial fraction of subjects typically reveals discount rates increasing in time horizon, (ii) the variability of discount rates across different commodities and geographical regions, and (iii) the coexistence of non-stationarity and dynamic consistency in experimental data. Our key predictions remain intact under bounded rationality and partial asset integration, and the mechanism is easily distinguished from hyperbolic preferences and optimistic bias.

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1 Introduction

Many of the most important choices we make involve alternatives with consequences materializing at different points in time. Prominent examples are how much to save for retirement, when to pay off debts or in which training to invest. Understanding what drives these choices is paramount for predicting individual behavior and market outcomes. In particular, the design of incentive mechanisms, information programs or optimal defaults preventing individuals from undersaving or taking other actions which are detrimental to their own future wealth or health needs to be based on a sound knowledge of where and how to intervene.

This paper makes three key contributions: First, it is - to the best of our knowledge - the first study on intertemporal choice investigating and formalizing the interaction between constraints and expectations. Previous research (reviewed below) focussed either on liquidity constraints or expectations, but did not examine the important interplay between these two dimensions. Introspection suggests that expectations are possibly most relevant for agents facing severe constraints. Indeed, this is exactly what our formal derivation substantiates. Second, we demonstrate that the interaction of constraints and expectations is key for understanding the joint (non-)occurrence of anomalously appearing behavior in lab and field data. More specifically, it appears that departures from exponential discounting along the dimensions time, outcome, sign and context are strongly interrelated. The present literature lacks an elaborate examination of these interactions. Third, we obtain a number of new and testable predictions beyond the standard anomalies widely discussed in the literature. An example comprises our predictions on domain-dependency of discounting behavior. In particular, we derive predictions about how discount rates and departures from exponential discounting differ between monetary outcomes, real effort and primary rewards if our mechanism is at play. As we argue below, these predictions match well to recent empirical findings on intertemporal choice.

Our work is motivated by a number of puzzling findings apparently inconsistent with exponential discounted utility, the canonical economic model in this domain. Indeed, most solutions proposed in the literature addressing these issues suffer from fundamental shortcomings: They typically address one or two anomalies only, impose nonconventional restrictions on preferences, and, thus fail to predict a series of important stylized facts as well as interactions between them. Hyperbolic preference models (Ainslie, 1975; Herrnstein, 1981; Mazur, 1987; Laibson, 1997; Harris and Laibson, 2001), for instance, capture excessive short-run discounting only. These models, however, do not provide an explanation for why individual behavior differs from standard predictions in many other important ways.

Empirical evidence documents the following findings. First, aggregate-level behavior departs in systematic ways from exponential discounting (Loewenstein and Thaler, 1989; Loewenstein and Prelec, 1992; Frederick, Loewenstein, and O'Donoghue, 2002). Quite robust anomalies are that discount rates lie far beyond market interest rates, decline in time horizon and in outcome magnitude, and are larger for gains as compared to losses. A coherent explanation encompassing all these findings is still missing.

Second, quantitative results, such as the magnitude of estimated discount rates or the extent of departures from exponential discounting, vary tremendously across and within studies (see e.g. Frederick, Loewenstein, and O'Donoghue (2002)).¹ This seems puzzling as most studies reporting quantitatively distinct results are based on similar experimental designs and are conducted among similar cohorts, i.e. subjects that are relatively homogenous with respect to their wealth, age and education. Related to this finding, important behavioral differences are documented among studies involving monetary and primary outcomes and among studies conducted across different geographic regions. Specifically, discount rates substantially differ between wealth and health outcomes (Chapman (1996), among others) as well as between money and effort outcomes (Augenblick, Niederle, and Sprenger, 2015). While some studies (e.g. Reuben, Sapienza, and Zingales (2010) and Ubfal (2016)) find strong and significant correlations between monetary and primary outcomes, others do not (e.g. Chapman (1996)). Regarding regional differences, Tanaka and Munro (2014) document that farmers living in less favorable agro-climatic zones discount future rewards more steeply than those living in more favorable zones. Taken together, these results indicate that particular characteristics of the good in question (such as the possibility to intertemporally reallocate units of it or the existence of markets) are detrimental factors driving cross-commodity heterogeneity in time discounting.

Third, longitudinal studies show that behavior is not as temporally stable as the prevalent preference models predict (Airoldi, Read, and Frederick, 2012). This indicates that intertemporal choices are influenced by factors other than deep preferences. Indeed, in a recent experimental study, Halevy (2015) finds that static preference reversals cannot fully explain dynamic inconsistencies. His data suggests that a considerable fraction of subjects change behavior over the passage of time in a way not predicted by models imposing particular restrictions on the discount function.

Finally, many studies report a substantial fraction of subjects exhibiting discount rates increasing in time horizon (Read, Frederick, Orsel, and Rahman, 2005; Sayman and Öncüler, 2007; Airoldi, Read, and Frederick, 2012; Abdellaoui, Attema, and Bleichrodt, 2010; Epper, Fehr-Duda, and Bruhin, 2011; Abdellaoui, Bleichrodt, and l'Haridon, 2013; Attema, Bleichrodt, Gao, Huang, and Wakker, 2016).² So far, it is not clear whether this behavior is due to mistakes, traits or possibly rational reasons.

The present paper addresses the question to what extent the above behavioral patterns can be rationalized within the bounds of standard economic theory. We build on the following intuition: When rational economic agents attempt to sustain a smooth consumption path, but are prevented from doing so because they only hold limited liquid assets and cannot borrow money at a reasonably low rate, their expectations about future income flows can affect their intertemporal choices in important ways. Opting for new alternatives materializ-

¹Frederick, Loewenstein, and O'Donoghue (2002) report discount rates ranging from -6 percent per annum to infinity.

²Related findings are reported by Frederick (1999); Rubinstein (2003); Read, Airoldi, and Loewe (2005) and Attema, Bleichrodt, Rohde, and Wakker (2010).

ing at dates when consumption is expected to be comparatively low permits these agents to reach consumption paths which better measure up to their goals (i.e. their preferences). Put differently, due to the fact that near-present low consumption periods cannot be smoothed away, the marginal utility of allocating new consumption earlier exceeds the marginal utility of allocating new consumption later.

That limited access to liquidity can amplify discount rates is supported by the empirical literature. In a study conducted among rural households in developing countries, Holden, Shiferaw, and Wik (1998) find that liquidity-constrained households show much higher discount rates than households not facing such constraints. Pender (1996) measures discount rates in rural India, and argues along similar lines: Exorbitantly high real interest rates charged by moneylenders generate binding credit constraints, which in turn can explain individuals' excess impatience. More recently, Dean and Sautmann (2016) report evidence consistent with the idea that credit constraints confound measures of time preferences. Using data from a panel study in Mali, they find that measured marginal rates of substitution are not very informative about underlying time preferences by means that partial credit constraints can drive a substantial wedge between behavior and deep preferences. A previous version of this manuscript (Epper, 2010) documents related evidence for two samples of Swiss university students: Students being cash constrained during the time of the experiment discount future rewards much steeper than those who are not.³ Recently, Carvalho, Meier, and Wang (2016) report very direct support for the mechanism we motivate here. They examine intertemporal choices of US low-income households before and after payday. While subjects in their sample face considerable shortage of liquidity, this shortage is more pronounced before than after receiving the pay check. Furthermore, subjects also exhibit a significant present bias before payday, but not shortly afterwards. This finding is perfectly consistent with the predictions we obtain here: Liquidity constraints together with positive income expectations (i.e. the expected arrival of the pay check) can generate excessive short-run discounting.⁴

The novelty of the present paper is that we study how liquidity constraints together with income expectations can unravel anomalies and a series of other puzzling findings in intertemporal choice. We argue that new alternatives are not evaluated in full isolation, as it is usually assumed in the empirical (and, in particular, the experimental) literature, but that there are situations where subjective income expectations are reflected in individuals' in-

³Epper, Fehr-Duda, and Schubert (2011) find comparable results for a sample of the broad Swiss german-speaking population.

⁴Liquidity constraints have also been found to be instrumental in explaining empirical regularities in consumer behavior. Deaton (1991), for example, shows that such constraints can produce more realistic consumption patterns than models abstracting from constraints. Related to this, Kaplan and Violante (2014) demonstrate that even agents holding substantial illiquid assets may behave like poor agents in that they reveal marginal propensities to consume close to one (or, in the words of Kaplan and Violante (2014), they are "wealthy hand-to-mouth" consumers). About one-third of all US households appear to be "hand-to-mouth" consumers, of which about two-thirds can be considered as wealthy (i.e. they hold significant illiquid assets) and about one-third can be considered as poor (i.e. they hold zero illiquid assets). Wealthy hand-to-mouth types can emerge when there are substantial transaction cost to liquidate illiquid assets, there are reasonably higher rates of return for illiquid assets or consumer credit comes at a high cost. Our model is compatible with Kaplan and Violante (2014)'s findings as we are only imposing restrictions on the *liquidity* agents have access to, but not on their overall wealth.

tertemporal choices. Indeed, there is evidence that narrow bracketing (Rabin and Weizsäcker, 2009) fails in the kind of choice situations we study here (see e.g. Epper (2010); Dean and Sautmann (2016); Ubfal (2016)). Epper (2010) (Section 3), for example, shows that subjects partially integrate experimental income with their out-of-lab consumption plans and that this happens even when stakes are comparatively small.

We present the following insights. First, we show that all the allegedly anomalous patterns naturally arise for liquidity-constrained, relatively impatient⁵ agents with positive rational expectations. In fact, and in contrast to existing explanations, all the “anomalies” are predicted to be closely intertwined with each other. As a result, our model prognosticates strong interaction effects among various violations of exponential discounted utility. In addition, behavior that appears to be hyperbolic from a static point of view, is not necessarily (time-)inconsistent from a dynamic point of view. Consequently, and in line with recent empirical evidence (Halevy, 2015), our findings challenge the view that static and dynamic preference reversals are one and the same.⁶ Heterogeneity in the constraints agents face and the expectations they hold can provide a reason for why subjects with otherwise fairly homogenous characteristics substantially differ in their intertemporal choices.

Second, our approach is first to provide a rationale for so far unexplained behavioral patterns documented in the literature. As time passes, the agents life and job situation is subject to change as is the economic environment. The agents’ opportunities to access liquidity and their consumption plans will therefore vary over the life cycle. Our approach can therefore provide a justification for the apparent dynamic instability (“time variance”) of choice behavior. Moreover, if an agent expects her income to substantially decline in the not so distant future (e.g. due to an anticipated temporary period of unemployment), but is unable to accumulate sufficient liquid assets to smooth away the upcoming low-consumption periods, she will exhibit discount rates increasing in time horizon. The distribution of exponential, hyperbolic and reverse-hyperbolic types in the population may therefore be largely governed by the liquidity constraints subjects face and the expectations they hold at the time of decision.

Third, we relax the assumption of perfect rationality to obtain a possibly more plausible model of intertemporal choice. We pursue two routes: In a first extension, we discuss the possibility that agents only partially integrate new consumption opportunities with their existing plans. There are two knife-edge cases: (i) The model we motivated before, i.e. the case where all new incomes are fully integrated with existing plans (*perfect asset integration*), and (ii) a model where all plans outside of the scope of the current decision situation are completely ignored (*narrow bracketing*). In the former case, choices will typically be more informative about subjective constraints and expectations than preferences, while, in the latter

⁵By “relatively impatient” we mean that the pure rate of time preferences exceeds the market interest rate (see Section 2 for further elaborations).

⁶Through the same channel, our approach can also explain violations of (time-)separability in intertemporal choice. Using revealed preference tests, Echenique, Imai, and Saito (2015) find that only about 52% of all subjects in the Andreoni and Sprenger (2012a) data and only about 43% of all subjects in the Carvalho, Meier, and Wang (2016) data have preferences which are rationalizable by time-separable utility. Similar findings are reported by Ericson and Noor (2015), who find that separable discounted utility is rejected for 68% of all subjects in their sample. These subjects exhibit magnitude-dependent discounting (but see our discussion later).

case, they will be fully informative about preferences. Empirical evidence suggests that reality lies somewhere in between these two extremes (see e.g. Epper (2010), Section 3). This poses challenges for testing intertemporal choice models and, in particular, for recovering deep preferences from choice.

In a second extension, we explore the possibility that agents hold overly optimistic beliefs about the future, but do not (necessarily) face liquidity constraints. This alternative explanation for a link between subjective expectations and time discounting is motivated by the finding that subjects often have optimistically biased beliefs when it comes to assessing future life events, such as future income flows (Weinstein, 1980, 1987; Dominitz, 1998; Armor and Taylor, 2002). Similar to agents with hyperbolic preferences and contrary to agents in the above rational model, such agents' behavior will be dynamically inconsistent, however. Recent research also hints that the same mechanism may explain why people suffer from self-control problems. Nordgren, van Harreveld, and van der Pligt (2009) find that people often overestimate their capacity for impulse control, leading them to overexpose themselves to temptations.⁷

There are ways to distinguish between competing explanations of anomalously-appearing intertemporal choices, i.e. hyperbolic preferences, optimistically biased beliefs and liquidity constraints together with positive income expectations, and we point at some possible directions to do so.

The present paper is related to recent contributions analyzing the role of constraints and expectations in intertemporal choice. The following paragraphs briefly distinguish our work from others. We first review the literature on constraints and then introduce several relevant papers investigating the role of expectations. Dean and Sautmann (2016) examine the effect of credit constraints on elicited time preferences.⁸ While their empirical findings can be seen as supportive evidence for the mechanism we motivate here, their theoretical analysis differs in several important ways from ours. First, in contrast to the present work, Dean and Sautmann (2016) do not attempt to rationalize anomalies in intertemporal choice within the standard model. In fact, they *assume* that preferences are quasi-hyperbolic. Our approach differs in that we obtain hyperbolic discounting as a *result*, given that certain conditions about the agent's access to liquidity and her expectations are met. Second, Dean and Sautmann (2016)'s primary interest lies on income and expenditure *shocks*, while we are mainly concerned with how news are integrated into existing plans.⁹ Relevant in our case is the *anticipated* change in future income. Indeed, our most central findings are the result of an interplay between liquidity constraints and the agent's income expectations. What motivates our approach is direct empirical evidence reporting strong correlations between expected income and discount rates.

⁷There may be rational reasons for the lack of self-control. Agents are required to exert willpower in order to resist temptation. If they (temporarily) only possess limited cognitive resources, they may not be able to exhibit the self control they need. The degree of (rational) inattention may therefore depend on the actual cognitive load the agents face. For other approaches viewing self control behavior as rational see Thaler and Shefrin (1981) or Fudenberg and Levine (2006).

⁸Pender (1996), Cubitt and Read (2007), and Meier and Sprenger (2010) argue along similar lines.

⁹For experimental evidence on the effect of income shocks on time discounting see Haushofer, Schunk, and Fehr (2013).

Epper (2010), Section 3, for example shows that students' out-of-lab income expectations are predictive for discount rates revealed in the laboratory. Finally, Dean and Sautmann (2016) and the present paper can be distinguished in the way they model constraints. Dean and Sautmann (2016) follow Harris and Laibson (2001) and model constraints by letting the *interest rate* to depend negatively on actual savings, i.e. by assuming that borrowing gets more expensive the more credit the agent is demanding. In contrast, we build on assumptions found in the household finance literature (e.g. Deaton (1991)), and assume the interest rate to be exogenously given and the agent's liquid wealth to be (weakly) positive.¹⁰ In our case, liquidity constraints become binding when the agent's liquid assets become relatively small, but the *pure rate of time preference* exceeds the real interest rate.

Ambrus, Asgeirsdottir, Noor, and Sandor (2015) provide experimental evidence for the hypothesis that income expectations influence elicited discount rates. They find that subjects with constant income expectations exhibit less present bias than subjects with non-constant income expectations. Together with the results presented in Carvalho, Meier, and Wang (2016) and Epper (2010), this is possibly the most clear evidence we can find for expectations determining discounting behavior. Contrary to the present paper, Ambrus, Asgeirsdottir, Noor, and Sandor (2015) do not attempt to provide a *unifying* explanation for the behavioral patterns we seek to explain in this paper. However, the method they propose to compensate for the confounding effect of income expectations on discount rates may be particularly useful for testing the mechanism we outline here.

Along similar lines, Noor (2009) and Gerber and Rohde (2015) point at issues when measuring discount rates in the presence of background consumption changing over time. They derive practical implications for eliciting time preferences. Both papers conclude that preference reversals can be produced by changes in baseline consumption. However, they both remain silent about the origin of non-constant consumption plans. The present paper fills this gap and thus complements their work. We seek to uncover the mechanism generating the "anomalies" reported in the literature, while this previous strand of research mainly focuses on describing one particular anomaly and providing valuable tools for experimentalists to control for potential bias in elicited time preferences.

The remainder of this paper is structured as follows. Section 2 introduces our theoretical model. Section 3 derives predictions under the assumption of perfect rationality and compares these predictions to well-documented empirical findings. Section 4 discusses some relevant extensions and implications. In particular, it covers the case of bounded rationality, biased beliefs and possibilities to discriminate between alternative mechanisms. The case of stochastic consumption plans is deferred to the appendix. Section 5 concludes.

¹⁰Once again, we say nothing about illiquid assets, and, hence, our approach is fully compatible with Kaplan and Violante (2014).

2 Model

This section introduces our basic model. We first focus on behavior of an agent with rational expectations who only holds few liquid assets and is limited with respect to her borrowing opportunities. The particular situation we are interested in is how this rational agent evaluates new alternatives by integrating them into her existing consumption plan. We remain completely within standard economic theory and follow standard preference assumptions (see e.g. Fishburn and Rubinstein (1982) and Manzini and Mariotti (2009)).

The agent we consider is characterized by the following preferences. First, the agent has constant impatience, i.e. for any two points in time with fixed temporal distance she has the same propensity to exchange later for earlier consumption, *ceteris paribus*. Under this assumption, the agent attributes weight $d(t) = \delta^t$, with $\delta \in (0,1]$, to future consumption utility, where d is the discount function and $\eta = -\ln(\delta)$ her (constant) pure rate of time preference. Strotz (1955) shows that such preferences are a necessity for dynamically consistent behavior.¹¹ We further assume that the agent is *relatively impatient*, i.e. that her rate of time preference η exceeds the real interest rate r . Only if this condition is satisfied, the agent has a need for additional liquidity sooner rather than later and, hence, she will demand credit or prefer to dissave from her (liquid) assets (see Deaton (1991)). Second, the agent has an aversion towards consumption fluctuations, i.e. she favors less variable consumption paths over more variable consumption paths, *ceteris paribus*. Her preference over consumption quantities is captured by an instantaneous utility function u satisfying standard regularity conditions (Inada, 1963).¹² Comparatively more concave utility functions imply a stronger preference for consumption smoothing.¹³

The agent has a consumption plan. This plan consists of a sequence of expected future consumption spendings. To form a plan, the agent uses her current information about future income. We assume rational expectations, i.e., on average, predicted income \hat{y}_t coincides with realized income y_t at any given point in time t , such that $\mathbb{E}[\hat{y}_t - y_t] = 0$.¹⁴ Three types of income expectations are distinguished: An agent who does not expect her income to change within the relevant time horizon is said to hold *constant income expectations*. Similarly, an agent who expects her income to substantially rise within the relevant time horizon is said to hold *positive income expectations*. The reverse is the case for *negative income expectations*.

Borrowing and saving permit the agent to transfer income back and forth in time. Likewise, an agent may dissave from her assets during periods with low income, but save during periods with high income. This allows her to sustain a smooth consumption path even if

¹¹As we will see later, such preferences are not sufficient to establish dynamic consistency, however.

¹²Viz. the utility function fulfills the following conditions: (1) $u(0) = 0$, (2) continuous differentiability, (3) $u' > 0$ (monotonicity/non-satiation) and $u'' < 0$ (concavity), (4) $u'(0) = \infty$ and $u'(\infty) = 0$.

¹³Comparatively more concave in terms of an Arrow-Pratt-type measure $-xu''(x)/u'(x)$. Note that utility embodies a very similar preference property in atemporal settings, where a comparatively more concave utility function expresses a stronger aversion towards variability of outcomes across *states* (instead of time).

¹⁴More precisely, the actual realization of consumption then deviates from expectations by some symmetric, independent random error, i.e. an error term which has an expected value of zero, is i.i.d. and uncorrelated with the information the agent holds at the relevant point in time.

income is subject to variation. Consequently, if access to liquidity is not restricted, expected short-term income changes should have little effect on actual consumption spendings.¹⁵ It is only the expected lifetime income, but not its variation, which governs current consumption in this case (Modigliani and Brumberg, 1954; Friedman, 1957).

Predictions differ fundamentally when the agent is limited with respect to her possibilities to access sources of liquidity, either because she faces liquidity constraints or because market frictions limit her scope of action.¹⁶ In what follows, we analyze behavior of a relatively impatient agent who is neither permitted to borrow, nor is endowed with substantial liquid assets. Nonetheless, as liquidity constraints only hamper borrowing, but not saving, such an agent may still overcome expected future low-income periods. Her limited borrowing opportunities together with the sparse liquid assets she holds, however, prevent her from smoothing away any enduring low-income period preceding a substantial rise in future income. As a result, consumption will respond asymmetrically to expected short-term income changes. The agent's expectations about changes in future income thus contain information about her behavior only if income expectations are positive, but not if they are negative or constant.¹⁷

That liquidity constraints are relevant is motivated by the following facts. First, there exists evidence that a substantial fraction of the population in both, rich and poor countries is affected by such limitations (Zeldes (1989), Deaton (1991), Kaplan and Violante (2014)). Estimates from the Panel Study of Income Dynamics (PSID), for example, suggest that about 20% of U.S. households are credit constrained (Hall and Mishkin, 1982; Jappelli, 1990).¹⁸ It is striking that most empirical evidence on anomalies in intertemporal choice is based on experiments conducted among students or among inhabitants of developing countries, i.e. with subjects who are likely to be exceptionally exposed to constraints. For obvious reasons, we can also expect that those subjects with the most immediate need for liquidity will also be those with the highest likelihood to select into studies promising monetary rewards. Second, there is direct empirical support for constraints affecting time discounting and consumption behavior. Holden, Shiferaw, and Wik (1998), for instance, find that liquidity-constrained households in developing countries exhibit much higher discount rates than households not facing such

¹⁵Similar predictions arise for a patient agent with $\eta \leq r$ even if there are limited borrowing opportunities (see Deaton (1991) for an elaborate discussion). For $\eta = r$, Schechtman (1976) and Bewley (1977) show that, under i.i.d. or stationary stochastic income processes, consumption converges to the mean of income even if liquidity constraints are present, permitting the agent to reach a perfectly smooth consumption path. Similarly, if $\eta < r$, the agent will save indefinitely. As Deaton (1991) formulates it: "[...] *saving, not borrowing, is [such an agent's] main concern*" (p.1225, text in brackets added).

¹⁶An example are high transaction cost which exacerbate the possibility to liquidate illiquid assets (see e.g. Kaplan and Violante (2014)).

¹⁷Contrarily, if a poor agent is neither permitted to borrow nor to save, responses are predicted to be symmetric. Her consumption plan then perfectly reflects her income expectations, or, in other words, there will be a comovement of consumption and income.

¹⁸Epper, Fehr-Duda, and Schubert (2011) conduct experiments with Swiss university students and a representative sample of the Swiss German-speaking population. Self-reports indicate that 44.8% of the university students and 9.7% of the general population were cash-constrained during the time of the survey. Related to this, Kaplan and Violante (2014) find that about one-third of all US households in the Survey of Consumer Finances hold close to zero liquid assets. These households thus exhibit marginal propensities to consume close to one.

constraints. Related findings are reported in Pender (1996); Epper (2010); Epper, Fehr-Duda, and Schubert (2011); Dean and Sautmann (2016); Carvalho, Meier, and Wang (2016) (see our literature review in Section 1). Others (Altonji and Siow, 1987; Shea, 1995; Drakos, 2002) find that (aggregate) consumption moves asymmetrically with predictable changes in income, a pattern in line with liquidity constraints, but not myopia.

We now derive our formal model. To do this, we build on standard exponential discounted utility theory (Samuelson, 1937; Fishburn and Rubinstein, 1982). We do, however, introduce liquidity constraints. Our basic assumptions closely follow those imposed by Deaton (1991).¹⁹ The agent chooses a consumption allocation which maximizes her total expected discounted consumption utility such that

$$\max_{(c_0, \dots, c_T)} \mathbb{E} \left[\sum_{\tau=0}^T \delta^\tau u(c_\tau) \middle| I_0 \right], \quad (1)$$

where $\mathbb{E}[\cdot | I_0]$ is her expectation conditional on the information I available at the decision date $\tau = 0$, c_τ is her *planned consumption* at period τ , and T is her planning horizon. In period $\tau + 1$, the agent's (liquid) assets are $w_{\tau+1} = (1 + r)(w_\tau + y_\tau - c_\tau)$, where r is the real interest rate and y_τ her exogenous (net) labor income earned in period τ . The borrowing restriction takes the form $w_\tau \geq 0$. The agent is only allowed to consume out of cash inflows from her "cash-on-hand" $w_\tau + y_\tau$. That is, consumption c_τ is bounded from above at $w_\tau + y_\tau$ and from below at zero, such that $0 \leq c_\tau \leq w_\tau + y_\tau$. We shall further assume that w_τ is relatively small. The nonnegative, but small liquid assets the agent maintains are only intended to insure herself against transitory and small negative income shocks. If not stated otherwise, it will therefore be convenient to presume that the agent can sustain a constant consumption path before and after a substantial and anticipated rise in income. This is possible because small income fluctuations are easily smoothed away by consuming out of the small buffer amount the agent holds.²⁰

Our primary interest lies in how new, i.e. previously unanticipated, alternatives are integrated into the agent's existing (ex-ante) consumption plan once they become available. Thereby, we restrict our attention to singular and certain cash in- or out-flows (henceforth referred to as outcomes) of which the agent learns at time $\tau = 0$. An alternative is a dated outcome x_t , where x is the outcome amount and t is the outcome date.²¹ Information about liquidity constraints is contained in the consumption plan. As limited access to liquidity makes it impossible to transfer future income to earlier dates positive income expectations will be reflected in improving baseline consumption plans. Nevertheless, as saving is still

¹⁹Similar assumptions can be found in Schechtman (1976), Bewley (1977), among others.

²⁰Small fluctuations of (disposable) income can be expected to occur more regularly than substantial changes or shocks in income or wealth. A forward-looking agent may therefore anticipate such small fluctuation and hold a sufficiently large buffer-stock to smooth them away.

²¹In the typical intertemporal choice experiment, subjects have to choose between smaller sooner and larger later outcomes. Commonly, it is impossible for the subject to anticipate these outcomes prior to entering the lab or study as she does only have limited information about what is going to happen in the experiment, what size the payoffs will have and when they eventually will be paid out.

allowed, this will not be the case for negative income expectations.²² Consequently, liquidity constraints limit the agent's possibilities to adjust her ex-ante consumption plan during the evaluation of new alternatives.

Figure 1 illustrates how expected income is transformed into an ex-post consumption plan in the presence and absence of liquidity constraints, respectively. The figure's annotations give a detailed description of this process.

To further illustrate this formally, consider an agent who is indifferent between two new alternatives, a present outcome z_0 and a future outcome x_t .²³ Suppose that she fully integrates z_0 and x_t into her ex-ante consumption plan (c_0, \dots, c_T) . The indifference $z_0 \sim x_t$ can then be expressed by

$$u(c_0 + z_0) + \mathbb{E} [\delta^t u(c_t) | I_0] = u(c_0) + \mathbb{E} [\delta^t u(c_t + x_t) | I_0] . \quad (2)$$

Note that only the two time periods $\tau = 0$ and $\tau = t$ appear in Equation 2. For all other time periods $\tau \notin \{0, t\}$, the consumption remains the same irrespective of which alternative would be chosen, i.e. these expected discounted period utilities cancel out from Equation 2. By rearranging the above equality, we obtain

$$u(c_0 + z_0) - u(c_0) = \mathbb{E} \left[\delta^t \left(u(c_t + x_t) - u(c_t) \right) \middle| I_0 \right] , \quad (3)$$

where the left-hand side of the equation denotes the utility increment from increasing consumption by z today, and the right-hand side of the equation denotes the utility increment from increasing consumption by x at t . If not stated otherwise, we set $c_0 = u(c_0) \stackrel{!}{=} 0$, and denote the change in baseline consumption relative to current consumption by $g_t = c_t - c_0$. This is motivated by the fact that typical intertemporal choice data will not permit identification of absolute levels of consumption (but see later).

Under these assumptions, the agent values any newly available dated outcome x_t relative to the expected change of consumption over the relevant time horizon. The dated outcome x_t is thus evaluated by

$$U_0(x_t) = \mathbb{E} \left[\delta^t \left(u(g_t + x_t) - u(g_t) \right) \middle| I_0 \right] . \quad (4)$$

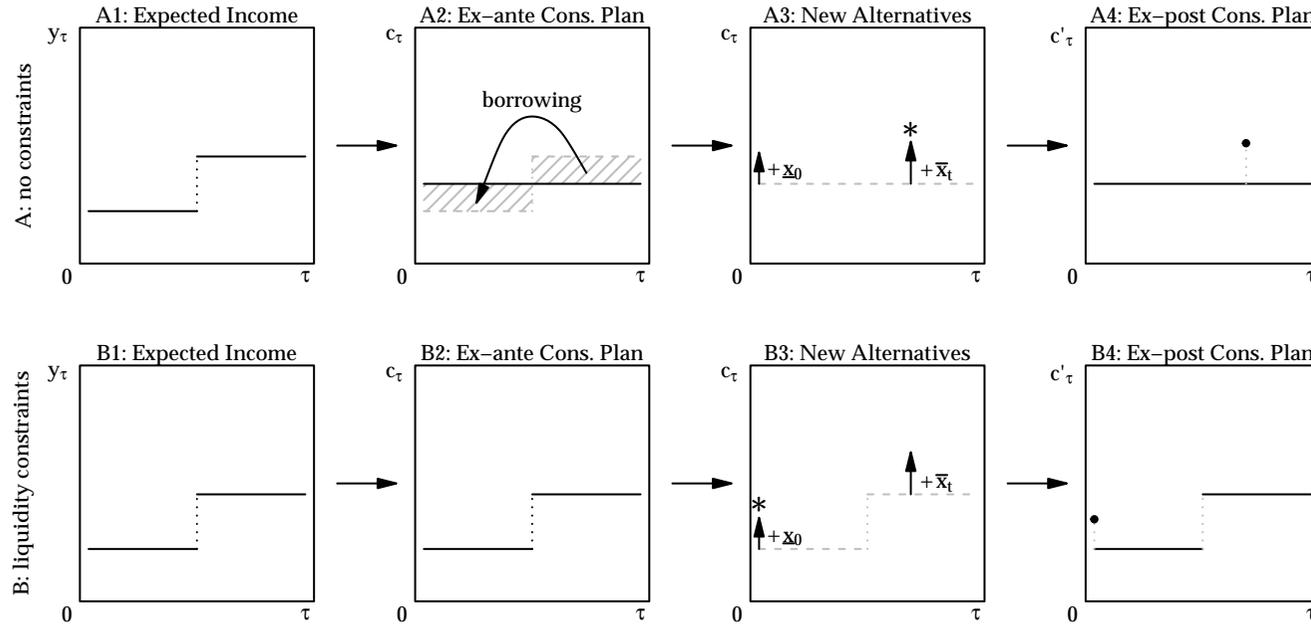
The availability of new alternatives opens up new possibilities for the agent to, at least partly, overcome her scarce liquid assets and her limited borrowing capabilities. Intuitively, a liquidity-constrained agent with the above preferences and positive income expectations can make the huge gap between low and high baseline consumption periods smaller by allocating new outcomes at earlier dates where baseline consumption is expected to be comparatively low. Due to concavity of instantaneous utility, marginal utility of consumption is larger at periods where planned consumption is low compared to when it is high.²⁴ This result directly

²²An agent with substantial negative income expectations requires sufficiently many opportunities to put money aside during high-income periods. Only if this is the case, she will be able to smooth away the anticipated low-consumption periods (see later).

²³In this paper, we only consider binary choices. However, the model is easily extended to the case where choices involve larger sets of alternatives.

²⁴As for example Browning and Crossley (2001) (page 4) phrase it "[...] within the life-cycle framework, smoothing does not mean keeping consumption or expenditures constant - far from it. Rather, smoothing means that agents try to keep

Figure 1: Illustration - Income Expectations and Consumption Plans



For illustrative purposes, we assume that the agent possesses a sufficiently small amount of liquid assets. The agent's preferences are characterized by a constant pure rate of time preference slightly above the real interest rate and a strictly concave instantaneous utility function. Here, we explore the case, where the agent expects her income to rise within the relevant time horizon. The upper **Row A** depicts the situation where the agent has full access to the credit market. **Panel A1** pictures her positive income expectations. In **Panel A2**, the agent borrows against future labor income, and, hence, increase here short-run consumption in exchange to longer-run consumption. The resulting consumption plan is smooth (solid black line). **Panel A3** introduces the choice between a smaller sooner payment \underline{x}_0 and a larger later payment \bar{x}_t . The agent opts for the payment at t (marked by $*$). **Panel A4** plots the resulting consumption path, which has a peak at t , but is constant otherwise. The lower row **Row B** depicts behavior in the presence of liquidity constraints. **Panel B1** pictures the positive income expectations. As borrowing is not possible, these income expectations are fully reflected in the ex-ante consumption plan (**Panel B2**). Being confronted with the same two payments as in the upper row, the agent opts for \underline{x}_0 (**Panel B3**). Intuitively, this is the case, as marginal utility from increasing consumption at $\tau = 0$ is larger than marginal utility from increasing consumption at $\tau = t$. Choosing the smaller sooner payment is a direct consequence of a preference for consumption smoothing. In other words, the availability of the new alternatives can be seen as a device to (partly) overcome the limited access to the capital market. **Panel B4** plots the ex-post consumption plan, with a peak in the present, and a pattern reflecting the positive income expectations afterwards.

follows from Equation 4. Note that the relatively low quantity of cash-on-hand together with the limited access to credit and the agent's relative impatience implies that, if there is no uncertainty, she does not have an incentive to spread consumption over the remaining low-income periods. Rather, she will consume the entire amount at the point in time it materializes (see e.g. Deaton (1991)), i.e. she will exhibit a marginal propensity to consume out of new cash inflows of one, given that the outcomes are sufficiently small.

3 Predictions

In this section, we present the model predictions and contrast them with existing empirical evidence. To do so, we first predict the present equivalent \hat{z}_0 making the agent indifferent to the dated outcome x_t , such that $\hat{z}_0 \sim x_t$, using the model we introduced previously. We then calculate the imputed annual discount rate $\hat{\eta}$ from the equality $\hat{z}_0 = e^{-\hat{\eta}t} x_t$, such that

$$\hat{\eta} = -\frac{1}{t} \ln \left(\frac{\hat{z}_0}{x_t} \right). \quad (5)$$

Note that this procedure closely follows the empirical literature, in which discount rates are commonly inferred from tradeoffs between smaller sooner and larger later outcomes. It therefore permits a direct comparison of our model's predictions with actual observations documented in this literature.

We make a number of assumptions which facilitate our analysis. First, if not stated otherwise, we assume isoelastic utility. We discuss the effect of alternative utility specifications at a later stage. Second, for expositional simplicity, we also presume that there is no uncertainty about the agent's (ex-ante) consumption plan, i.e. that (c_0, \dots, c_T) is *a priori* known. This permits us to drop the expectation operator. However, as we discuss in the appendix, this assumption does not impose much of a limitation as any uncertain consumption plan can be replaced by its certainty-equivalent consumption plan leaving the qualitative results we obtain intact.

We are primarily interested in comparing behavior of a liquidity-constrained, relatively impatient agent holding positive income expectations with that of an agent who does either not face such constraints, because she holds sufficient liquid assets, is able to borrow at a low rate, is sufficiently patient, or has constant income expectations. Remember that according to our rational model, income expectations only contain information about the agent's behavior if she is limited with respect to her access to liquidity.

The next subsections are devoted to illustrate the dependency of predicted discount rates on time horizon, outcome magnitude and outcome sign. Further predictions are sketched out in a special part at the end of this section.

the marginal utility of money constant over time, which may involve quite variable expenditures."

3.1 Time Horizon

We first analyze the impact of time horizon on predicted discount rates. Our model's predictions are a direct result of the basic intuition provided earlier: Liquidity-constrained agents with positive income expectations allocate new cash inflows at dates where baseline consumption is expected to be comparatively low. In this situations, low-consumption periods precede high-consumption periods. As a consequence, liquidity-constrained agents are predicted to exhibit a comparatively more pronounced preference for more immediate payoffs than agents who do not face such constraints or expect to be able to maintain a constant baseline consumption path, *ceteris paribus*. The effect diminishes as the delay grows larger, however, resulting in predicted discount rates declining hyperbolically in time horizon and converging towards constant long-run rates. Intuitively, the marginal utility of allocating consumption in low baseline consumption periods is higher than the marginal utility of allocating consumption in high baseline consumption periods. This, again, is a direct result of an aversion towards consumption fluctuations. For a liquidity-constrained, relatively impatient agent positive income expectations therefore generate a markup on otherwise constant discount rates. The extent of the markup is predominantly driven by how large the agent expects her income to rise within the relevant time horizon and by how large the outcomes are.

To see this, consider the following formal derivation. According to our model, the predicted present equivalent \hat{z}_0 making the agent indifferent to the dated outcome x_t is equal to

$$\begin{aligned}\hat{z}_0 &= u^{(-1)}\left(\mathbb{E}\left[\delta^t\left(u(c_t + x_t) - u(c_t)\right)\middle|I_0\right] + u(c_0)\right) - c_0 \\ &= u^{(-1)}\left(\delta^t\left(u(g_t + x_t) - u(g_t)\right)\right),\end{aligned}\tag{6}$$

where the second line takes into account our assumptions that the consumption plan is *a priori* known and that $c_0 = u(c_0) \stackrel{!}{=} 0$. Recall that information about the agent's liquidity constraints is already contained in the ex-ante consumption plan. For our results to hold, it is important that the expected rise in consumption occurs sometimes in between the point in time the decision is made and the point in time the dated outcome is materializing.

Calculating the discount rate inferred from our model by inserting the predicted present equivalent in Equation 6 into Formula 5, differentiating with respect to t and rearranging, gives

$$\frac{\partial \hat{\eta}}{\partial t} = -\frac{1}{t} \left[\underbrace{-\frac{1}{t} \ln\left(\frac{\hat{z}_0}{x_t}\right)}_{\hat{\eta}} - \underbrace{(-\ln(\delta))}_{\eta} \underbrace{\frac{u(\hat{z}_0)}{\hat{z}_0 u'(\hat{z}_0)}}_{1/\varepsilon} \right].\tag{7}$$

The interpretation of the term in square brackets is straightforward. It is the difference between the agent's *behavior*, i.e. the predicted discount rate $\hat{\eta}$, and her *deep preferences*, i.e. the

product of the pure rate of time preference η and the reciprocal of the elasticity of the utility function $1/\varepsilon$. The subsequent comparative static analysis illustrates the basic mechanism driving short-run excess discounting.

For $u'' < 0$ and $g_t > 0$, the fraction \hat{z}_0/x_t decreases as g_t increases, ceteris paribus. This is the case since larger g_t s produce stronger discounting of prospective consumption utility, i.e. due to concavity of u , the utility difference $u(g_t + x_t) - u(g_t)$ becomes smaller the larger the expected change in consumption g_t gets. As δ^t is a constant with $\delta^t \in (0, 1]$, and the function $u^{(-1)}$ is strictly monotonically increasing in its argument, the predicted discount rate $\hat{\eta}$ increases as consumption expectations g_t grow larger. Under isoelastic utility, the fraction $1/\varepsilon$ is constant. Given all this, the difference in square brackets is positive and, hence, it holds that $\partial \hat{\eta} / \partial t < 0$. In plain terms, predicted discount rates decline in time horizon, with higher consumption expectations producing a more pronounced gap between behavior and deep preferences.

A number of additional findings directly emerge from Equation 7. First, anticipated baseline consumption changes do not take effect under utility linear in consumption, simply because g_t cancels out in $\hat{\eta}$ and $1/\varepsilon = 1$. Similarly, no effect is predicted for $g_t = 0$ if u is isoelastic. As a consequence, both, a preference for consumption smoothing and an improving consumption plan are necessary to produce discount rates decreasing in time horizon. Second, in the long run, predicted discount rates are not distinguishable from constant rates even if $g_t > 0$, since $\lim_{t \rightarrow \infty} \partial \hat{\eta} / \partial t = 0$. Third, contrary to hyperbolic discounting models, decreasing discount rates are not a consequence of the agent's deep preferences, but they are the result of her income expectations and the borrowing limitations preventing her from smoothing away the preceding low-consumption periods. In our model, the size of the effect therefore depends on anticipated consumption changes g_t , a dependency not predicted by hyperbolic preference models. Fourth, it directly follows from Equation 7 that there is an interaction between decreasing discount rates and the magnitude effect. Holding g_t constant, predicted discount rates decline less strongly in time horizon the larger x becomes, ceteris paribus.²⁵ A more elaborate discussion of this effect follows in the next subsection.

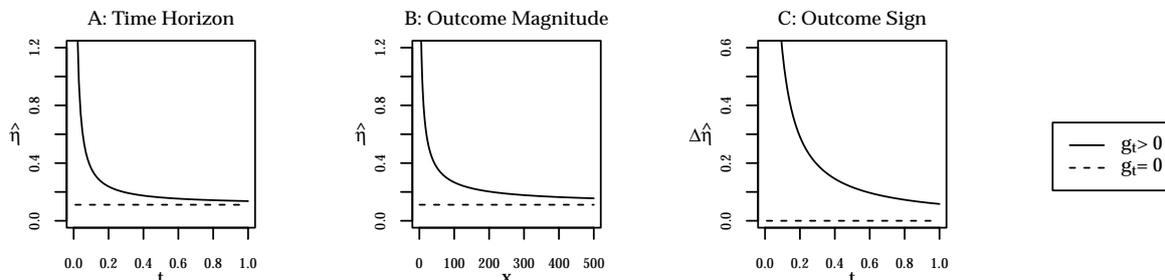
As a remark, consider the case where the elasticity of utility is not constant. There are two cases for which this applies: Either the utility function does not have the power/logarithmic form, or the nonzero (absolute) level of baseline consumption c_0 is an argument of the otherwise isoelastic utility function (see the first line in Equation 6). An example for the former case is exponential utility. In the latter case, c_0 takes the role of an 'anti-index' of concavity of u (see Wakker (2008) for details). Both, exponential utility and nonzero absolute levels of consumption are able to generate discount rates decreasing in time horizon as well as the other "anomalies" without necessitating the mechanism motivated here. However, within plausible parameter ranges, such utility specifications are only able to produce effects which are *too small* to reconcile the empirical findings.²⁶ Furthermore, the direct empirical evidence

²⁵It is a direct consequence of this finding that violations of separability between the evaluation of outcomes and time are predicted by our model.

²⁶Elaborate calibration results are beyond the scope of this paper. Interested readers are referred to Noor (2011)

linking liquidity constraints and income expectations to elicited discount rates (see the literature review at the beginning of this manuscript) clearly speaks against the conjecture that non-constant elasticity of utility is the main driving force behind observed violations of exponential discounted utility.

Figure 2: Graphical Depiction of Model Predictions



The three panels depict our model's key predictions. **Panel A:** For constant consumption plans, $g_t = 0$, the predicted discount rate $\hat{\eta}$ is constant in time delay t (dashed horizontal line). Improving consumption plans, $g_t > 0$, produce a markup on these discount rates. The predicted discount rates decline in time horizon (solid hyperbolic curve). **Panel B:** For constant consumption plans, $g_t = 0$, the predicted discount rate does not depend on outcome magnitude x holding t fixed (dashed horizontal line). Improving consumption plans, $g_t > 0$, lead to excessive discounting for small stakes (solid hyperbolic curve). This effect diminishes as stakes grow larger. **Panel C:** This figure plots the difference between discount rates predicted for gains and discount rates predicted for losses, i.e. $\Delta\hat{\eta} = \hat{\eta}^+ - \hat{\eta}^-$ against time delay t . No meaningful gain-loss asymmetry is predicted for $g_t = 0$. For $g_t > 0$, however, the asymmetry between gains and losses is most pronounced for short delays t , but it diminishes when increasing the delay t . Similar results obtain when plotting the difference in domain-specific discount rates against outcome magnitude x . *Side note:* Graphical illustrations assume that the agent expects an increase of (baseline) consumption sometimes between the point in time the decision is made and the point in time the outcome materializes. The following parameters were used to construct the graphs: $\eta = 10\%$, $u(z) = z^{0.9}$, $g_t = 2$; Panel A: $x = 40$; Panel B: $t = 1/12$; Panel C: $x = 1$.

Panel A of Figure 2 illustrates our findings. It plots predicted discount rates for both, an agent with an improving consumption plan (solid curve) and an agent with a constant consumption plan (dashed line). As can be seen, improving consumption plans induce decreasing discount rates, ceteris paribus. Short-run discount rates therefore appear much larger than long-run discount rates. Despite the agent having a constant pure rate of time preference, her expectations drive a significant wedge between her time preferences and her discounting behavior. Very similar results can be obtained for losses, i.e. consumption reductions rather than consumption increases.²⁷ However, due to concavity of the utility function, predicted

who provides results for such a calibration exercise for the case of the magnitude effect. Similar arguments can be found in the domain of choice under risk, where the concavity of the utility function is not sufficient to produce meaningful degrees of loss aversion (see Rabin (2000) and others). Related to this, a previous version of this paper (see Epper, Fehr-Duda, and Bruhin (2011)) provides estimation results for a constraints-expectations model. It shows that comparatively modest expectations in the range of several hundred USD per month are sufficient to rationalize the average departures from exponential discounting.

²⁷Obviously, this either requires total consumption to be nonnegative, or an appropriate normalization of the utility function.

discount rates converge faster towards constant rates in the loss domain.²⁸ We come back to this result in Section 3.3.

The predictions of the liquidity-constraints model accord well with the empirical evidence on intertemporal choice. For monetary rewards, Thaler (1981) observes annualized median discount rates of several dozens to hundreds percent. Discount rates decline sharply in time horizon. Similar findings are reported in Benzion, Rapoport, and Yagil (1989) and many other studies (see e.g. Redelmeier and Heller (1993); Chapman and Elstein (1995); Chapman (1996); Pender (1996); Frederick, Loewenstein, and O'Donoghue (2002); Epper, Fehr-Duda, and Bruhin (2011); Halevy (2015)).²⁹ Empirical evidence also supports our long-run predictions. Pender (1996), for example, finds that discount rates far away from the present cannot be distinguished from constant ones. Similar results are reported for negatively signed outcomes (Benzion, Rapoport, and Yagil, 1989; Chapman, 1996). Most studies find much lower discount rates in this domain, a finding corresponding well with our predictions.³⁰

Two important properties of intertemporal preferences have been widely discussed in the literature. The first property, *stationarity*, says that preferences should not revert when moving two dated outcomes along the time axis while keeping the temporal distance between them fixed. Stationarity is a static property. It involves comparison of two pairs of differently dated outcomes viewed from the same point in time. Formally, stationarity is defined as follows:

Definition 1 (Stationarity) *If $x_t \sim z_{t+\lambda}$, then $x_s \sim z_{s+\lambda}$, $\forall x, z$, and $0 \leq t < s, \lambda > 0$.*

According to this property, preferences are fully determined by the temporal distance λ . In other words, changing the delay of the two outcomes relative to the point in time the decision is made should not alter preferences. It has been shown that stationarity is necessary for exponential discounted utility (Strotz, 1955; Fishburn and Rubinstein, 1982), and that preference reversals of the form $x_t \succ z_{t+\lambda}$ and $x_s \prec z_{s+\lambda}$ are the critical property instantiated by hyperbolic preference models (Prelec, 2004; Bleichrodt, Rohde, and Wakker, 2009).

The second property, *dynamic consistency*, says that preferences should not revert when reevaluating a choice at some later point in time. As its name says, dynamic consistency is a

²⁸Due to concavity, subtracting x units from c_t leads to a utility loss which is in absolute terms larger than the utility gain when adding x units to c_t . In utility terms, losses loom larger than gains.

²⁹Some studies do not find support for discount rates declining in time horizon. Examples are Andreoni and Sprenger (2012a) and Augenblick, Niederle, and Sprenger (2015). The authors argue that decreasing discount rates may be a consequence of not holding transaction costs constant across different payment dates or not making all payments equally reliable. Clearly, these are important factors which can produce or amplify decreasing discount rates. Related research (Epper, Fehr-Duda, and Bruhin, 2011), however, finds that, even when controlling for equal transaction costs and taking all efforts to make payments equally reliable, there is still a majority of subjects exhibiting such behavior. We hypothesize that the differences between these conflicting results may be due to sample differences (richer vs. poorer subjects, i.e. due to the mechanism described in the present paper) or due to the different elicitation methods used in these studies (Andreoni and Sprenger (2012a) and Augenblick, Niederle, and Sprenger (2015) use convex time budgets, while most other studies cited in this paper use binary choices or choice lists). Carvalho, Meier, and Wang (2016)'s findings speak in favor of the former (i.e. our) explanation (see our discussion in the introductory section). Recent research (Balakrishnan, Haushofer, and Jakiela, 2015) also indicates that the immediacy of the "sooner" payments is crucial for detecting present bias.

³⁰It should be noted, however, that it is usually not possible to implement real losses in laboratory experiments, as it is normally not permitted for researchers to extract a net amount of money from participants (for an exception see Etchart-Vincent and l'Haridon (2011)). The results in this domain may therefore be influenced by framing decisions.

dynamic property. It involves comparison of the very same pairs of dated outcomes viewed from different points in time. Formally, dynamic inconsistency is defined as follows:

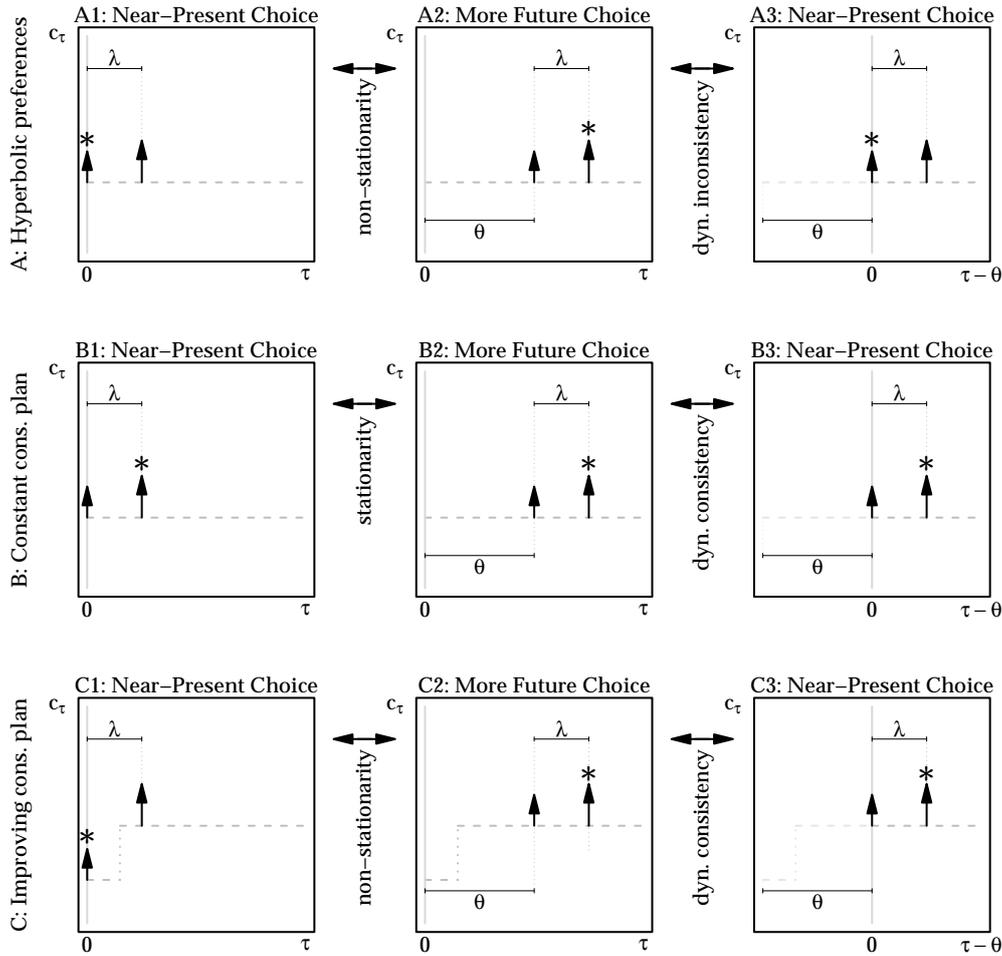
Definition 2 (Dynamic consistency) *If $x_s \sim z_{s+\lambda}$, then $x_{s-\theta} \sim^\theta z_{s+\lambda-\theta}$, $\forall x, z$, and $\lambda > 0$, $0 < \theta \leq s$,*

where \sim^θ denotes indifference at the later point in time, θ time units after the original assessment. According to this property, preferences should be fully determined at one single point in time. Put differently, the point in time from which the two dated outcomes are evaluated should not affect preferences.

The large part of the hyperbolic preference literature makes no distinction between violations of stationarity and violations of dynamic consistency (see e.g. Thaler (1981) or Augenblick, Niederle, and Sprenger (2015)). Indeed, it is often (implicitly) assumed that static preference reversals fully inform about dynamic preference reversals, i.e. that time-inconsistencies are the result of hyperbolic preferences. This view is valid if and only if the agent's deep hyperbolic preferences completely describe her choice behavior, i.e. if preferences are hyperbolic and completely stable over time, and no factors other than the agent's deep preferences (such as income or wealth shocks) influence behavior. If this is the case, only the temporal distance relative to the point in time the decision is made will matter, and we typically expect that $x_t \succ z_{t+\lambda}$, $x_{t-\theta} \succ^\theta z_{t+\lambda-\theta}$, but $x_s \prec z_{s+\lambda}$. Row A of Figure 3 illustrates this choice pattern. Again, in the absence of any changes in the environment, hyperbolic preferences predict that non-stationarities generate dynamically inconsistencies (see also Rohde (2010)).

Contrary to this theoretical prediction, empirical research suggests that static preference reversals cannot (fully) explain dynamic preference reversals. Airolidi, Read, and Frederick (2012) report results from a longitudinal study, which permits both, a test of stationarity and a test of dynamic consistency. Their findings indicate that factors outside of deep preferences play an important role, and they demonstrate that violations of stationarity should be uncoupled from violations of dynamic consistency. Halevy (2015)'s experimental setup permits a clean and separate test of the two properties. He finds that only a fraction of subjects with time-consistent preferences eventually exhibits stationary preferences.

Our model provides an explanation for this evidence: Still assuming that deep preferences are characterized by a constant pure rate of time preference and strictly concave instantaneous utility, we obtain the following predictions: First, if a constant consumption plan can be retained, we predict neither static nor dynamic preference reversals (see row B of Figure 3). In this case, both stationarity and dynamic consistency are satisfied. Second, if the consumption plan is improving within the relevant time horizon, the model predicts static, but no dynamic preference reversals (see row C of Figure 3). Intuitively, anticipated changes of baseline consumption are tightly linked to the *calendar date* at which they occur. Assuming that no new information regarding income or access to liquidity becomes available over the passage in time, the consumption plan remains the same. In contrast, it is the *temporal delay* with regard to the point in time the decision is made which generates the non-stationarities and dynamic inconsistencies in hyperbolic preference models. Row C and A in the figure visualize these



The figure depicts static and dynamic preference reversals under three different setups. *Stationarity* posits that revealed preferences do not revert between column 1 and column 2. *Dynamic consistency* posits that revealed preferences do not revert between column 2 and column 3.

Row A shows predictions for a hyperbolic preferences model with narrow bracketing. The agent exhibits excessive short-run discounting and prefers the smaller sooner payment over the larger later payment when confronted with the choice between two near-present outcomes (indicated by * in **Panel A1**). She prefers the larger later payment, however, when the two outcomes are shifted into the future while keeping their temporal distance constant (* in **Panel A2**). Her preference reverts when reconsidering this choice at some later point in time (**Panel A3**), and she again favors the smaller sooner over the larger later payment. It is only the delay relative to the point in time the decision is made which matters for such preferences. As a consequence, hyperbolic preferences predict a tight link between violations of stationarity and violations of dynamic consistency.

Row B illustrates predictions for standard preferences where the agent can retain a smooth consumption path. The agent prefers the larger later payment (* in **Panel B1**). The same is the case when moving the payments into the future while keeping the temporal distance between the two fixed (* in **Panel B2**). **Panel B3** depicts the situation where the agent is reconsidering the choice she made in the past (see Panel B2). Behavior does not change in this setup. An exponential preferences model with narrow bracketing yields equivalent predictions.

Row C shows predictions for standard preferences and an improving consumption plan. The agent picks the smaller sooner payment (* in **Panel C1**) which permits her to, at least partially, compensate for the relatively low consumption periods in the near present. Confronted with the choice between the same two payments in the more distant future (**Panel C2**), she opts for the larger later option (as in Panel B2). The agent *appears* to violate stationarity. However, she won't revert her choice when reconsidering it at some later point in time (**Panel C3**) as her consumption plan remains stable. This result requires that news becoming available over time do not systematically change the agent's plan. Our approach therefore untangles the link between non-stationarity and dynamic inconsistency postulated by the hyperbolic discounting literature.

predictions and show that they differ in important ways between the model outlined in the present paper and hyperbolic preference models. Finally, if the ex-ante consumption plan is constant over the relevant time horizon, but new information about substantial and systematic changes in future income or liquidity constraints becomes available over the course in time, the model is able to predict dynamic preference reversals (not depicted in Figure 3).³¹ Our model can therefore generate all possible combinations of static and dynamic preference reversals, and it remains an empirical question whether it is able to explain the results documented by Halevy (2015). It is also noteworthy that our model predicts time-inconsistencies as a result of a preference for consumption smoothing (i.e. concavity of the utility function) rather than a feature of the discount function. Directly related to this finding, our approach is able to generate violations of separability between outcomes and time (for recent nonparametric evidence see Echenique, Imai, and Saito (2015) and Ericson and Noor (2015)). Once again, this is the case as the consumption plan is linked to the calendar date, or, put differently, the argument of the utility function does not only depend on the magnitude of the outcome, but also on the point in time it arrives.

Our approach can also provide an explanation for why many studies find a substantial fraction of subjects exhibiting discount rates increasing in time horizon (see for instance Read, Frederick, Orsel, and Rahman (2005); Sayman and Öncüler (2007); Airoldi, Read, and Frederick (2012); Abdellaoui, Attema, and Bleichrodt (2010); Epper, Fehr-Duda, and Bruhin (2011); Abdellaoui, Bleichrodt, and l’Haridon (2013); Attema, Bleichrodt, Gao, Huang, and Wakker (2016)). If a(n) (relatively poor) agent expects her income to decline substantially in the not so distant future, but is unable to accumulate sufficient liquid assets to overcome the anticipated low-consumption periods, she will exhibit increasing discount rates. The story is the same as before, but here the agent is limited with respect to the accumulation of liquid assets. The predicted effects are once again asymmetric, but they have the opposite sign. Intuitively, opting for new alternatives materializing during future low-consumption periods may help her to partially overcome these limitations. As a result, predicted discount rates will lie below the agent’s pure rate of time preference. To the best of our knowledge, our approach is the first providing a possible explanation for this puzzling finding. We hypothesize that agents with close to no liquid assets and substantial negative expectations are most likely to exhibit such behavior.³²

3.2 Outcome Magnitude

Our second set of predictions concerns the magnitude effect, i.e. the empirical regularity that smaller outcomes are discounted at a higher rate than larger ones. This effect is a direct consequence of the agent’s temporal allocation of new cash inflows at dates where marginal

³¹To provide an illustrative example: An agent may prefer a healthy apple over a bar of chocolate right after a meal, but prefer the chocolate over the apple after a longer time of no caloric intake. While such revealed preferences appear to be dynamically inconsistent, they are barely the result of time-inconsistent *deep preferences*, but more plausibly the consequence of a change in the constraints (i.e. the actual caloric requirement).

³²These agents will exhibit a marginal propensity to consume out of their cash-on-hand close to one (see also Kaplan and Violante (2014) for further elaborations).

utility is largest. As a given absolute change in baseline consumption has a higher impact on smaller outcomes compared to larger outcomes, *ceteris paribus*, predicted discount rates will decline in outcome magnitude. Intuitively, the larger the outcomes become relative to the anticipated increase in baseline consumption, the smaller the difference in marginal utility between the sooner and later allocations. The effect is largely governed by the marginal rate of intertemporal substitution between future and present consumption, and it is more pronounced the stronger baseline consumption is expected to grow, *ceteris paribus*. Negligible or very distant positive changes in baseline consumption are unlikely to result in significant magnitude-dependency, however.

To see this, consider the derivative of predicted discount rates $\hat{\eta}$ with respect to the outcome magnitude x :

$$\frac{\partial \hat{\eta}}{\partial x} = -\frac{1}{t} \left[\underbrace{\frac{\delta^t u'(g_t + x_t)}{u'(\hat{z}_0)}}_{\text{MRS}_{t,0}} \frac{1}{\hat{z}_0} - \frac{1}{x_t} \right]. \quad (8)$$

Recall that the predicted present equivalent is equal to $\hat{z}_0 = u^{(-1)}(\delta^t(u(g_t + x_t) - u(g_t)))$. The size of the magnitude effect largely depends on how much the agent expects her consumption to rise during the relevant time horizon, i.e. the size of g . The first term in square brackets, the product of the marginal rate of intertemporal substitution between future and present consumption $\text{MRS}_{t,0}$ and the reciprocal of the present equivalent $1/\hat{z}_0$, rises as g_t grows larger.

The following comparative static proves the magnitude effect and its dependence on g_t . Consider two dated outcomes with $\bar{x}_t > \underline{x}_t > 0$. $u' > 0$ implies that $\hat{z}_0(\bar{x}_t) > \hat{z}_0(\underline{x}_t)$, *ceteris paribus*. From $u'' < 0$ it follows that $(u^{(-1)})'' > 0$ and $\hat{z}_0'' > 0$. As a result, the discount fraction $\hat{z}_0(\bar{x}_t)/\bar{x}_t$ is larger than the discount fraction $\hat{z}_0(\underline{x}_t)/\underline{x}_t$. Since $\hat{\eta}_{\bar{x}_t} < \hat{\eta}_{\underline{x}_t}$, predicted discount rates decrease as x_t grows larger. Following the same argument, it is easily shown that the effect becomes more pronounced when g_t increases, holding x_t and all other things fixed.

Some additional results emerge from Equation 8. First, anticipated changes in baseline consumption do not take effect if u is a linear function of its argument, i.e. if marginal utility is constant. For this case, we thus predict no magnitude effect. Similar results are obtained for $g_t = 0$ and isoelastic u . In this case, the first term in square brackets reduces to $1/x_t$, which is the lower bound of the product in this term given that $g_t \geq 0$ and $u'' < 0$. Second, predicted discount rates converge to rates constant in outcome magnitude as x tends towards infinity, *ceteris paribus*, since $\lim_{x_t \rightarrow \infty} \partial \hat{\eta} / \partial x_t = 0$. Similarly, the magnitude effect diminishes as the time horizon grows towards infinity, since $\lim_{t \rightarrow \infty} \partial \hat{\eta} / \partial x_t = 0$. This result, once again, indicates that the two effects, the time horizon effect and the magnitude effect, are closely intertwined, a prediction not made by prevalent models proposed in the literature (we discuss

some exceptions below).

Panel B of Figure 2 illustrates these findings. For a liquidity-constrained, relatively impatient agent with positive income expectations, predicted discount rates are substantially larger for small outcomes as compared to large outcomes, but they decline as the stake size increases (solid curve). This is not the case for an agent who is able to sustain a smooth consumption path (dashed line). Such an agent is not sensitive to changes in outcome magnitude, but reveals predicted discount rates constant in stake size. We also predict a magnitude effect for losses, i.e. consumption reductions. Again, due to concavity of the utility function the effect is predicted to be less pronounced in this domain. Details follow in the next subsection.

Our predictions dovetail nicely with the empirical evidence provided by numerous studies (Thaler, 1981; Loewenstein, 1987; Benzion, Rapoport, and Yagil, 1989; Holcomb and Nelson, 1992; Raineri and Rachlin, 1993; Shelley, 1993; Green, Fristoe, and Myerson, 1994; Green, Myerson, Lichtman, Rosen, and Fry, 1996; Kirby and Marakovic, 1995; Kirby, 1997; Kirby, Petry, and Bickel, 1999; Halevy, 2015). For positive outcomes, Thaler (1981) not only finds that median discount rates decrease in outcome magnitude, but also that *“subjects’ actions are closer to the normative model, the larger are the stakes”* (p.206). Similar results, although much less pronounced, are reported for losses (Thaler, 1981; Benzion, Rapoport, and Yagil, 1989; Shelley, 1993; Chapman, 1996).

Despite its prevalence and its relevance for understanding choices over small and large stakes, most discounting models, such as hyperbolic preference models, fail to predict a magnitude effect. Some exceptions exist. Loewenstein and Prelec (1992) propose a descriptive reference-dependent model which imposes specific assumptions on the elasticity of the utility function to accommodate the magnitude effect. Noor (2011) postulates a magnitude-dependent discount factor and derives hyperbolic preferences as a special case. Epper and Fehr-Duda (2015b) demonstrate that the presence of a per-period risk to get less than a promised reward can produce magnitude-dependent discounting. Holden (2014) presents a theory of “mental zooming” which accommodates both the magnitude effect and hyperbolic discounting. Noor (2011), Epper and Fehr-Duda (2015b) and Holden (2014) are all able to produce interaction effects between magnitude and delay, although they all consider very different mechanisms generating the effects.³³

The magnitude effect and its interaction with delay is relevant as it can provide a better understanding of durable goods purchase decisions. When different durables are evaluated, the comparatively small, recurring costs often get too little weight compared to the relatively large, up-front purchasing price. Importantly, in such situations, magnitude dependency is hardly separable from time horizon effects and gain-loss asymmetries. Another example is provided by Noor (2011): In sequential bargaining games, equilibrium predictions differ fundamentally in the presence of the magnitude effect as compared to when it is absent. Under magnitude-dependent preference, there exist multiple equilibria, in some of which the first mover receives a smaller share even though she is the more patient of the two players. Ericson

³³None of these papers does analyze the sign effect and the list of additional patterns we discuss below, however.

and Noor (2015) also make clear that violations of separability, as generated by magnitude dependency, are crucial when extrapolating from small stake evidence to large stake behavior.

3.3 Outcome Sign

So far, we found that subjective income expectations can induce asymmetric discounting behavior with respect to differently signed outcomes. The sign effect, i.e. the regularity that gains are discounted more heavily than losses, is thus predicted for typical agents in our model. The intuition is as follows: Due to concavity of the instantaneous utility function, sacrificing consumption always hurts more than increasing consumption by the same absolute quantity yields pleasure. It follows from the previously examined effects that the difference between domain-specific discount rates diminishes as the time horizon or the absolute outcome magnitude becomes larger. As a result, behavior is predicted to be more symmetric between gains and losses the further in the future the outcomes materialize and the larger these outcomes are, *ceteris paribus*.

The formal derivation straightforwardly follows from the above results. The sign effect emerges from applying Jensen's inequality to the gain-loss difference of predicted discount rates $\Delta\hat{\eta} = \hat{\eta}^+ - \hat{\eta}^-$. Under typical assumptions, it holds that $\lim_{t \rightarrow \infty} \partial [\Delta\hat{\eta}] / \partial t = 0$ and $\lim_{x \rightarrow \infty} \partial [\Delta\hat{\eta}] / \partial x_t = 0$.

Panel C of Figure 2 depicts our predictions by plotting the difference of the domain-specific discount rates against time horizon. Agents with improving consumption plans exhibit asymmetric discounting behavior (solid curve). The asymmetry between gains and losses is largest for short time horizons, but diminishes the farther in the future the outcome materializes. Symmetric behavior is predicted for an agent who can sustain a constant consumption path (dashed line). Similarly, the sign effect vanishes as stakes grow larger.

Empirical studies such as Yates and Watts (1975), Thaler (1981), Loewenstein (1987), Loewenstein (1988), Benzion, Rapoport, and Yagil (1989), MacKeigan, Larson, Draugalis, and Bootman (1993), and Chapman (1996), all find that gains are discounted more heavily than losses. Evidence also seems to suggest that this effect diminishes as the time horizon and the outcome magnitude grow larger (see e.g. the results reported in Thaler (1981)).

Most discounting models do not predict sign-dependency. A notable exception is Loewenstein and Prelec (1992).³⁴ In this reference-dependent model the sign effect arises by *ex-ante* assuming different elasticities of the utility function for gains and losses. Walther (2010) follows a different route and derives the sign effect as a consequence of disappointment aversion. Hyperbolic preferences alone, however, cannot generate a gain-loss asymmetry, simply because such predictions lie outside of the scope of the model. Nevertheless, a better understanding of this effect is crucial for interpreting choices where both, costs and benefits come into play, such as in investment decisions or the purchase of durables.

³⁴See Abdellaoui, Bleichrodt, and l'Haridon (2013) for an experimental examination of this model.

3.4 Further Predictions

Our model makes a series of additional predictions beside the stylized facts we discussed so far. Some of these predictions have, to the best of our knowledge, not yet been examined in detail so far.

The formal derivation of the above “anomalies” has shown that they are inextricably linked with each other. The reason for this is that they all originate from the agent’s allocation of new cash inflows at dates where baseline consumption is comparatively low. Put differently, they are all the result of one and the same mechanism. We therefore expect the extent of departures from exponential discounting with respect to time horizon, outcome magnitude and outcome sign to be strongly and positively correlated across subjects. We are not aware of any existing test of this prediction.

A few remarks are in order. First, note that we only predict interaction effects for shorter time horizons and modest stake sizes. If these conditions are not met, the two effects and their interaction should be rather hard to detect. This prediction has important implications for applications of discounting models. For instance, it suggests that exponential discounting may be a good approximation for evaluating strategies mitigating global warming even when financial resources are very limited and substantial technological progress can be expected. This is the case since such decisions are typically characterized by long time horizons and large stake sizes.

Second, that one common mechanism drives various distortions of discount rates can be exploited to falsify our model, and the very same prediction can also be used to test the model against alternative approaches. Hyperbolic preference models, for example, attribute decreasing discount rates to deep preferences, but fail to predict magnitude- and sign-dependency. The extended model introduced by Loewenstein and Prelec (1992) attempts to capture every single anomaly separately by imposing specific assumptions on the discount function and the utility function. These models therefore do neither predict an interaction between “anomalies”, nor violations of separability, whereas the approach proposed here does.

Third, we predict the “anomalies” to be most pronounced for agents with improving consumption plans, or, more specifically, for agents facing binding liquidity constraints and having positive income expectations. Earlier in this article, we reviewed some literature which is in line with this prediction. The dependence of discount rates on these factors can be used to test our model against models assuming narrow bracketing. Moreover, as a preference for consumption smoothing is crucial for predicting the “anomalies”, we also expect a correlation between the preference for consumption smoothing (i.e. the curvature of the utility function) and the degree of departures from exponential discounting.³⁵

As agents differ with respect to their preferences, their access to liquidity and their income expectations, the above mechanism may also provide an explanation for the huge variation within and between studies (as e.g. documented by Frederick, Loewenstein, and O’Donoghue (2002)). While most behavioral models assume that heterogeneity is solely driven by interper-

³⁵Note that our model does not predict the “anomalies” if utility is linear or convex.

sonal differences in deep preferences, our analysis reveals the importance of environmental factors and expectations for understanding the vast behavioral variation. Beside the direct evidence on the link between liquidity constraints, income expectations and time discounting listed in the first section of this paper, this could also explain why poorer subjects typically exhibit much higher discount rates (see e.g. Tanaka, Camerer, and Nguyen (2010)). Moreover, it also points at difficulties of using discount rates elicited from typical (laboratory) subjects to predict behavior of broader populations, or to even rely on such calibrations when formulating policy recommendations. Students, for example, are likely to exhibit stronger violations of exponential discounting than typical subjects from the general population. This is the case as the latter group typically had more opportunities to accumulate liquidity in the past and, hence, has less problems accessing fresh capital (see e.g. Epper, Fehr-Duda, and Schubert (2011) for some evidence).

The very same mechanism may also help to explain a number of related findings. First, there are well-documented differences in discount rates across regions. Tanaka and Munro (2014), for example, recognize that *“[r]isk attitudes and discount rates are not merely reflection of personal preferences but represent economic and other conditions of the individuals and households”* (p.152). They find that farmers in more favorable agro-climatic zones discount future outcomes less steeply than others. A plausible explanation for this finding is that these farmers accumulated more liquid wealth, and, as a result, that they are less likely to face liquidity constraints as compared to farmers operating in harsher environments. It would therefore be interesting to know whether farmers’ expectations indeed correlate with their discount rates, as predicted by our model. While it is evident that institutional factors, such as market frictions, underdeveloped capital markets or the lack of (micro-)credit, may explain differences of discount rates across developing and developed countries, we are not aware of elaborate studies examining the effect of these economics factors on intertemporal choices. A notable exception is Wang, Rieger, and Hens (2016) who find considerably more present bias in developing economies as opposed to developed economies. This finding can potentially be explained by the different development of financial and capital markets. Also, taking the GDP per capita as a proxy for wealth indicates a strong and significant correlation with present bias, which again is consistent with our theory.

Second, the very same argument may be put forward to explain differences in time discounting across commodities, such as between monetary and primary outcomes. Chapman (1996), for example, reports anomalies for both, choices involving money and choices involving health. However, she finds no evidence for a correlation of discount rates between the two domains. In addition to this result, average discount rates for health improvements typically exceed those for monetary gains (see e.g. Chapman (1996)). Related to this finding, Augenblick, Niederle, and Sprenger (2015) find only very limited evidence for decreasing discount rates for money, but a rather pronounced present bias in an effort task. The key for understanding these behavioral differences between monetary and primary rewards may be characteristics of the good (or bad) under consideration: Money is fungible and it is typically

relatively easily transferrable back and forth in time, assuming that the agent has unrestricted access to the capital market. This is not the case for health or real effort. While it is clearly not possible to buy or borrow quantities of health in exchange to money or future units of health, it also seems very unlikely that people in the Augenblick, Niederle, and Sprenger (2015) real effort experiments hired others to do their work. Choices involving primary outcomes may therefore appear very similar to choices involving monetary outcomes in the presence of (liquidity) constraints. Hence, it is plausible that differences in the characteristics of the commodities (e.g. the existence of markets or possibilities to intertemporally reallocate quantities of the commodity) and not deep preferences are the most fundamental drivers of the behavioral differences we observe. The evidence presented by Carvalho, Meier, and Wang (2016) is in line with this reasoning: The low-income household participants they survey exhibit present bias in a real effort task, irrespective of whether they are asked before or after the paycheck arrived. However, these participants only show present-biased behavior for monetary outcomes prior to the wage payment, but not afterwards. Obviously, receiving the salary only relaxes the constraint for money, but not the one for effort.

Many important questions remain yet to be answered. One is the question of causality. Specifically, do people end up in poverty because of the institutional deficiencies they face (e.g. the lack of a functioning credit/capital market or substantial transaction costs) or do they select into poverty because they are notoriously impatient? The present model suggests that the former mechanism may play the prevalent role. A better understanding of this causality is relevant for developing redistributive policies, for which it typically matters whether wealth inequality originates from preference heterogeneity or differences in (exogenous) constraints. Unfortunately, the existing empirical literature does not permit a unanimous conclusion on this causal link.³⁶

4 Discussion

We now present two model extensions and discuss some possible directions for distinguishing between different models. So far, we have demonstrated that many well-documented findings reported in the literature can be produced by fully rational agents with limited access to liquidity. In the next two subsections we weaken the rationality assumption in two ways, and obtain (possibly more plausible) conditions yielding qualitatively similar conclusions. The first extension (Section 4.1) relaxes the assumption of full asset integration. The second extension (Section 4.2) relaxes the assumption of rational expectations. A further extension is deferred to the appendix. It demonstrates that our predictions remain intact if the agent is uncertain about her future consumption opportunities, given that certain conditions regarding her expectations are met. The last part of this section (Section 4.3) outlines some key implications of two boundary cases of a generalized version of our model and compare these implications to those of hyperbolic preferences.

³⁶On the theoretical side, a possible future extension could be to formulate a model which fully endogenizes liquidity constraints and impatience.

4.1 Extension I: Partial Asset Integration

It is straightforward to extend the above model to accommodate partial asset integration. When doing so, the predictions remain qualitatively intact. However, as compared to the case of full asset integration, the effects turn out to be comparatively less pronounced. For the extreme case of narrow bracketing (i.e. no asset integration), we predict no departure from exponential discounting.

Consider the following (modified) valuation function:

$$U_0(x_t) = \mathbb{E} \left[\delta^t \left(u(\kappa g_t + x_t) - u(\kappa g_t) \right) \middle| I_0 \right], \quad (9)$$

where $\kappa \in [0, 1]$ denotes the (asset) integration parameter. This model contains two notable edge cases. First, $\kappa = 1$ represents the case where the agent performs *full asset integration*. In this case, the model coincides with the rational model we introduced before. Second, $\kappa = 0$ represents the case of *narrow bracketing*. In this case, the model collapses with the exponential discounted utility model used as a benchmark in most of the empirical work on time discounting. It essentially says that consumption opportunities outside of the current choice bracket or situation (e.g. the laboratory) are completely disregarded by the agent. We refer to the intermediate case with $\kappa \in (0, 1)$ as *partial asset integration*.

From Equation 9 it is straightforward to see that our predictions remain intact as long as $\kappa > 0$. However, putting less attention on changes in baseline consumption (i.e. a comparatively smaller integration parameter κ) also means that the “anomalies” will be less pronounced, *ceteris paribus*. This is the case since the term representing the anticipated changes, κg_t , becomes comparatively smaller if κ decreases.³⁷ It follows from Equation 9 that choice behavior will typically be more informative about subjective expectations than deep preferences if $\kappa > 0$. In contrast, observed behavior will fully reflect deep preferences if $\kappa = 0$.

Epper (2010), Section 3, presents some evidence for the model introduced in this paper, and shows that aggregate departures from exponential discounted utility can be explained to a large part by the rational liquidity-constraints mechanism proposed here. Partial asset integration may give a reason for the remaining gap between the rational model’s predictions and actual observations. However, it is important to understand that the results reported by Epper (2010), Epper, Fehr-Duda, and Schubert (2011), Ambrus, Asgeirsdottir, Noor, and Sander (2015), Dean and Sautmann (2016) and Carvalho, Meier, and Wang (2016) are not in line with narrow bracketing, as this would imply that out-of-study constraints and expectations were not correlated with choices in the experiment or survey.

4.2 Extension II: Bounded Rationality

There is a second mechanism through which subjective income expectations can translate into the above behavioral patterns, and this mechanism does not necessarily require liquidity constraints to be present nor does it require that the agent’s pure rate of time preference

³⁷In the appendix we show that uncertainty in the consumption plan can have very similar effects.

exceeds the market interest rate. Instead, this mechanism substitutes the rational expectation assumption by optimistically biased expectations. More specifically, an agent prone to optimistic bias expects her consumption opportunities in the future to be better than they actually are. Such biased beliefs can be easily accommodated by our model, and there is sound empirical evidence that they play an important role in the type of choice situations we consider here. Again, the key predictions remain intact when such considerations come into play, two notable exceptions being our predictions on dynamic consistency and potentially the dependence of discounting behavior on scarce liquidity.

Empirical evidence reports that subjects are often too optimistic when it comes to evaluating future life events (Weinstein, 1980, 1987; Armor and Taylor, 2002). They typically overestimate their future earnings (Dominitz, 1998) and their ability to resist future temptations (Nordgren, van Harreveld, and van der Pligt, 2009). For our setting, this means that future income is systematically overestimated, such that $\mathbb{E}[\hat{y}_t - y_t] > 0$ for any point in time t , where \hat{y} is the income predicted by the agent and y is the eventually realized income. In the most extreme case, an optimistically biased agent would completely ignore any possibility to intertemporally transfer income back and forth in time by saving, dissaving or borrowing, and, consequently, she would use new cash inflows for instantaneous consumption. If this is the case, income y is equivalent to consumption c , i.e. the marginal propensity to consume is equal to one.³⁸ Now set $c_0 = 0$, as baseline consumption is completely irrelevant for this agent. $g_t > 0$ in our original model (see Equation 4) can then be interpreted as the *optimistic bias*. As this optimistic bias will be strictly positive, the agent's beliefs about the future imply that marginal utility derived from consumption today is always larger than the marginal utility derived from consumption at some future point in time, irrespective of her pure rate of time preference. Even if there is no (rationally) anticipated increase in baseline consumption over the relevant time horizon, the key anomalies prevail. Nevertheless, there are some important exceptions as well as differences to the above rational model which allow to distinguish the two mechanisms. We discuss some of points in detail below.

The model in which the agent completely ignores any outside consumption plan or outside opportunity is likely too extreme. A more plausible model could be one which is close to the rational model above, but still allows for some degree of overoptimism. One way to incorporate this idea is to postulate that g_t does not fully measure rationally anticipated change in baseline consumption, but is instead a convex combination of rational anticipations and optimism. For example, let g_t in our original model be $g_t = \lambda h_t + (1 - \lambda)b_t$, where h_t measures the rational change in baseline consumption during the relevant time period and b_t measures optimism expressed in monetary terms. $\lambda \in [0, 1]$ denotes the weight the agent attributes to rational changes of consumption, and, hence, it can be interpreted as a rationality index. According to this extended model, optimistic bias can have the reverse effects of partial asset integration, and, hence, amplify the "anomalies" as long as $b_t > h_t$ and $\lambda \in [0, 1)$. In this case,

³⁸Note that the predictions we obtain are very different from the rational model, where the change in marginal utility is linked to the calendar date and not the temporal distance from the present. As we show below, there are ways to distinguish between alternative explanations for excessive short-run discounting.

non-stationarities are again coupled to dynamic inconsistencies. One application for which such an extended model could be useful is given by recent empirical evidence showing that schooling makes subjects' expectations more rational (Shenoy, 2015). We therefore expect λ to increase with schooling, and, consequently, dynamic inconsistencies to be more prevalent among subjects with lower education.

4.3 Further Remarks

There are ways to distinguish between different mechanisms generating anomalies in intertemporal choice. Here we outline some possible directions which empirical researchers may exploit. We focus on three models. First, our *rational model* in which departures from exponential discounting are generated by an interplay of liquidity constraints and (rational) income expectations. This model corresponds to the model we introduced in the main part of the paper (Section 2). Second, an *optimism model* in which these departures are generated solely by the agent's overly positive outlook. This model corresponds to the extension introduced in the previous subsection with $\lambda = 0$. The typical agent in this model abstracts from integrating new alternatives into her existing consumption plan. However, we suspect that, to some degree, the current economic situation may still affect overall optimism. Third, we compare these models' predictions to those of a *hyperbolic preference model* which attributes decreasing discount rates fully to deep preferences, i.e. a hyperbolic discount function. This model assumes narrow bracketing and, hence, independence of choice from constraints and expectations.

The three models make a series of distinct predictions and these predictions have different implications. The predictions permit researchers to discriminate among the three mechanisms, i.e. they enable an understanding of whether *preferences*, *beliefs* or *constraints* govern distortions of discount rates. The models' implications, on the other hand, are relevant for policy makers who want to implement measures counteracting actions with potentially negative welfare effects. It is noteworthy that the boundedly rational model introduced in the last subsection, i.e. the model with $\lambda \in (0,1)$, can make predictions somewhere in between the rational model and the optimism model. For instance, it can predict that both, stationarity and dynamic consistency are violated, despite the agent taking into account her shortage of liquidity. The boundedly rational model is easier to distinguish from hyperbolic preferences than the pure optimism model as it relaxes the bracketing assumption inherent in the latter two.

Both, the rational model and the optimism model, are able to predict the entire set of "anomalies" as well as their interactions. Contrary to this, hyperbolic preferences alone are not able to generate the magnitude effect, the sign effect and the other patterns we outlined in Section 3.4. Instead, this class of models predicts excessive short-run discounting only. However, there are two ways to extend hyperbolic preference models such that they can accommodate outcome- and sign-dependence. A first possibility is to impose additional restrictions on the utility function. Loewenstein and Prelec (1992) pursue this route and assume the utility

function to be more elastic for outcomes that are larger in absolute magnitude (generating the magnitude effect) and to be more elastic for losses than for gains (generating the sign effect). Still, as in this model all the anomalies are a result of separate preference assumptions, either affecting the utility function *or* the discount function this approach does not predict interactions between “anomalies”.³⁹ A second possibility to capture the magnitude and sign effect is to let the discount function to depend on outcome magnitude and outcome sign. Using a calibrations theorem, Noor (2011) argues that this route should be favored over explanations involving concavity of utility, and, hence, the Loewenstein and Prelec (1992) approach.⁴⁰ He axiomatizes a model in which the discount factor depends on the magnitude of outcomes and demonstrates that hyperbolic discounting is contained as a special case in this representation. It directly follows from this specification that his magnitude-dependent model also predicts an interaction between time horizon effects and magnitude effects. While Noor (2011) does not discuss the sign effect, his analysis could be extended in this direction.

The most often used hyperbolic preference models (see Frederick, Loewenstein, and O’Donoghue (2002) for a review) cannot reconcile the fact that a considerable fraction of subjects typically exhibits discount rates increasing in time horizon (see our literature review in Section 1). Bleichrodt, Rohde, and Wakker (2009) propose two discount functions which resolve this limitation. Such discount functions are very useful for descriptive purposes as they are flexible enough to characterize a wide range of discounting patterns. Once again, however, without imposing additional assumptions on the utility function or discount function, these discount functions cannot reconcile the other anomalies. In addition, it is still not clear where increasing discount rates originate from. Our model fills this gap and yields a series of novel and testable predictions. More specifically, our rational model predicts that such behavior is due to subjects holding relatively few liquid assets and expecting a decline in income during the relevant time horizon. The optimism model, on the other hand, predicts that pessimistic beliefs about the future drive such behavior.

Another possibility to distinguish the different models is to consider static and dynamic preference reversals. The three models make distinct predictions when it comes to stationarity and dynamic consistency, and when it comes to the link between these two properties. As we have seen before, the rational model uncouples non-stationarities from dynamic inconsistencies and it is able to generate various combinations of static and dynamic preference reversals. In contrast, these two kinds of preference reversals are one and the same in the optimism model and in the hyperbolic preference model. There is one notable exception, however: If the agent is aware of her dynamically inconsistent deep preferences or her notoriously optimistic beliefs, she might be willing to prevent dynamic preference reversals by commitment. A possible commitment strategy for sophisticated agents is to allocate wealth in illiquid assets in order to inhibit future selves from consuming “too much”.⁴¹ An agent

³⁹In other words, this model retains separability in the outcome and time domain.

⁴⁰Noor (2011)’s calibration critique affects Loewenstein and Prelec (1992), but not our approach (see our discussion in Section 3.2 and Footnote 26).

⁴¹While sophisticated agents gained some attention in the theoretical literature (see e.g. O’Donoghue and Rabin (1999)), there appears to be only very limited demand for commitment in both, real-world and experimental

in the rational model would never engage in such actions. Similar predictions arise for naive hyperbolic discounters or agents not aware of their systematically overoptimistic beliefs.

Experimental tests may also exploit the bracketing assumptions underlying the three models. The rational model predicts that variables outside of the actual choice situation matter and that newly available information is fully integrated into existing plans. We expect similar, but less pronounced responses for the optimism model, where the subject's mood and attitude may be influenced by economic factors or recent changes in the environment.⁴² Also, we suspect the degree of departures from exponential preferences to be correlated with optimism scores (such as those obtained by psychological questionnaires on optimism, see e.g. Scheier, Carver, and Bridges (1994)). The hyperbolic preference model considered here, however, relies on narrow bracketing and, hence, it does not predict a relationship between discount rates and outside factors. The empirical evidence we reviewed in the beginning of this paper suggests that there is indeed a correlation between discount rates and liquidity constraints, and between discount rates and income expectations. In particular, the results reported in Carvalho, Meier, and Wang (2016) and Epper (2010) indicate that an interaction between constraints and expectations is an important determinant of the patterns we observe. While we are not aware of any study exploring the correlation between discount rates and overoptimism, there appears to be strong evidence for the basic mechanism we motivate here. Future research may also make use of the fact that rational agents respond to new information about systematic changes in future income by integrating this information into their existing consumption plan. Experimenters may therefore recover the underlying consumption plan by confronting subjects with a sequence of intertemporal choices in which each single decision is being paid out.

The fact that distinct causal mechanisms are at work in the three models may help to identify the actual mechanism. Positive attitudes drive anomalously-looking behavior in the optimism model. Possibly more interestingly, the rational model and the hyperbolic preference model make opposing causal claims with regard to the link between decreasing discount rates and liquidity constraints. While notorious overconsumption due to hyperbolic preferences might expose agents to further shortages in liquidity and eventually guide them into poverty, the rational model argues that it is these limitations which ultimately produce hyperbolically-shaped discounting behavior. It is an important empirical question which mechanism is more relevant in explaining the co-occurrence of present bias and liquidity constraints. Our analysis led to a number of novel predictions, all of which are testable. For example, we predict that experimental manipulations of expectations and income or wealth shocks may help identifying the underlying mechanism.

After all, we believe that the role of the institutional environment cannot be neglected. Access to (micro-)credit, as an example, might assist agents to get out of (transitory) liquidity

settings (Laibson, 2015). Laibson (2015) argues that in many cases the cost of commitment are just too high, such that commitment does not pay off for the decision maker. Consistent with this conclusion, Augenblick, Niederle, and Sprenger (2015) only find low willingness to pay for commitment. Nevertheless a considerable fraction of subjects seems to be willing to commit if commitment comes at no charge.

⁴²For related results in the domain of choice under risk see Fehr-Duda, Epper, Bruhin, and Schubert (2011).

shortages which were caused by severe wealth shocks in their past. Facilitated access to liquidity might thus be crucial for rational agents to find their way out of poverty.⁴³ To provide another example, people are often reluctant to purchase energy-efficient durables. The temporal shortage of liquidity can be one reason for why they are not willing to consider the long-run cost savings of choosing more efficient alternatives (see e.g. Epper, Fehr-Duda, and Schubert (2011)). If this is the case, providing further information, e.g. in the form of energy labels, will not help encouraging energy-efficient investments. Providing alternative ways to finance the investments, on the other hand, could resolve the problem.

This brings us back to the question which of the three competing mechanisms is the most plausible. In the previous section, we reviewed evidence in favor of the rational model. We do not conjecture that this model explains all the available evidence, but suspect that a boundedly rational model with λ closer to one than zero is possibly best in line with the empirical findings we reviewed earlier. While there is ample evidence for optimistic bias, the nature of static and dynamic preference reversals clearly speaks in favor of the hypothesis that some aspects captured by the rational model are relevant.

If departures from exponential discounting are predominantly driven by optimism, then one may reason that it does not matter whether we model intertemporal choices as the outcome of *beliefs* (as in the optimism model) or as the outcome of *preferences* (as in hyperbolic preference models). However, our results demonstrate that hyperbolic preferences may only be suited well as a reduced form when the focus lies on excessive short-run discounting, but significant variation in outcome magnitude or changes in outcome sign can be ruled out. Further direct tests of the above mechanisms are ultimately needed before drawing a definite conclusion, however.

5 Conclusion

We introduced a unifying rationalization of anomalies in intertemporal choice. Subjective expectations can have substantial impact on agents' discounting behavior and they can drive systematic departures from exponential discounting beyond those predicted by alternative approaches. For this understanding, the interaction between liquidity constraints and expectations is key. We are, to the best of our knowledge, the first formalizing this interplay and investigating its implications.

In the first part of the paper, we explored the case of a fully rational agent with standard preferences, and we have shown that all the apparently puzzling behavioral patterns can naturally occur as the result of an interplay between liquidity constraints and income expectations. The assumptions required for our predictions to hold are plausible for typical participants in studies reporting the anomalies, and the mechanism is also operative when stakes are comparatively small. The empirical findings reported in the literature are in line

⁴³On the other hand, eased access to credit may guide hyperbolic agents into consuming "too much" (see e.g. Laibson (1997) who argues that financial innovation in the US in the form of the introduction of credit cards was instrumental for the overconsumption patterns we observe in data).

with our predictions and they provide very direct evidence for our model. Moreover, we also obtained a series of novel and testable predictions, which can be used to falsify our approach.

In the second part of the paper, we relaxed the strict rationality assumptions of our main model, and we have shown that our key predictions remain intact for the cases of partial asset integration and bounded rationality. There are many ways to distinguish hyperbolic preferences from optimistic bias and our rational mechanism, and this is important as all three models have very different economic implications.

Our approach differs from previous attempts to accommodate anomalies in intertemporal choice. For instance, the usual procedure in behavioral economics is to directly contrast observed behavior with standard preferences. In the domain of intertemporal choice, this means that elicited discount rates are compared to exponential preferences. Other important drivers of behavior outside of deep preferences are usually not explicitly considered, however. This can be problematic since the interaction of constraints and expectations can fundamentally distort discount rates. One can therefore not refute economic theories without considering all the primitives they build upon. Indeed, taking into account the characteristics of the commodities in question, such as the existence of markets and the possibility to intertemporally reallocate units of the good may be key for understanding differences in choices involving monetary and primary outcomes. Similarly, there are many behavioral patterns which are hard to reconcile by stable hyperbolic preferences alone. One example is people's reluctance to buy health insurance when they are in a good state of health, but their willingness to spend a very large part (or all) of their wealth for health care once they have diagnosed a life-threatening disease. It seems more plausible that this change in revealed preferences is driven by a change in constraints rather than the instability of deep preferences.

The fact that anomalously-appearing behavior is not necessarily driven by irrational types with time-inconsistent preferences, but might well be caused by fully rational, liquidity-constrained agents poses important challenges for applied economics and policy making. It suggests that constant discount rates are not the proper criterion for identifying rationality, and that it can be dangerous to attribute all departures from exponential discounting to exotic deep preferences. In addition, hyperbolic preferences and the rational liquidity-constraints model make opposing claims on why people end up in poverty or undersave for retirement. Policy makers should thus have an interest to uncover the actual mechanism at play before determining which measure to implement.

A Appendix

A.1 Extension III: Stochastic Consumption Plans

In the main text we assume that future consumption is *a priori* known. As shown in this appendix, our qualitative predictions remain robust if we allow for uncertainty in the consumption plan. This extension potentially leads a series of new comparative static results which are beyond the scope of the model in the paper.

There are two reasons for a stochastic component in future consumption. First, future income is always subject to some uncertainty.⁴⁴ For example, a student searching for a job does not exactly know the wage offers she will receive in the future. Similarly, many employment contracts contain a variable wage component, such as tips, bonuses or other performance-dependent compensations, making it impossible to perfectly predict future labor income. Furthermore, an employee may have a chance to get promoted and, hence, she may assign a positive likelihood to move into a higher wage bracket. Likewise, she may envisage the possibility of getting unemployed.

Second, unanticipated and substantial changes in wealth can alter the agent's access to liquidity. Examples are positive wealth shocks, such as inheritance, or negative wealth shocks, such as a large and unexpected loss of (uninsured) assets. The fear of wealth and income shocks should therefore be reflected in the agent's consumption plan, and, hence, her discount rate (see e.g. Haushofer, Schunk, and Fehr (2013) for some related empirical evidence).

There are different ways to incorporate uncertainty in our model. The possibly most straightforward approach is to recognize that, for any uncertain (ex-ante) consumption plan, there exists a certainty-equivalent consumption plan making the agent indifferent. Formally, suppose that for any point in time t , there is a stochastic consumption level $\tilde{g}_t = g_t + \varepsilon$ with $\mathbb{E}[\varepsilon | g_t] = 0$ and ε being independently and identically distributed noise. Then, the agent is indifferent between the degenerate consumption level (i.e. the certainty equivalent) q_t and the stochastic consumption level (i.e. the random variable) \tilde{g}_t , such that $q_t \sim \tilde{g}_t$. To make one further assumption explicit, we presume that both, risk aversion and consumption smoothing are entailed in the instantaneous utility function u . We do this for simplicity, and to demonstrate the effect the introduction of risk and risk aversion have on discounting behavior.

We now consider the situation where a newly available and certain alternative x_t is evaluated against the ex-ante consumption plan.⁴⁵ Under the above assumptions, the following equivalency holds:

$$U_0(x_t) = \delta^t \left(u(q_t + x_t) - u(q_t) \right) \equiv \mathbb{E} \left[\delta^t \left(u(\tilde{g}_t + x_t) - u(\tilde{g}_t) \right) \middle| I_0 \right]. \quad (10)$$

⁴⁴Even long-run contractual agreements are always to some degree incomplete.

⁴⁵We do not discuss the case where x_t is risky, as this has been done before. Epper and Fehr-Duda (2015b), Epper and Fehr-Duda (2015a) and Epper and Fehr-Duda (2016) analyze this situation theoretically, and Epper, Fehr-Duda, and Bruhin (2011) and Andreoni and Sprenger (2012b) provide empirical evidence. A more complicated, but potentially promising future direction is where both x_t and c_t are stochastic and their combination reduces the agent's risk exposure (i.e. one is a hedge against the other).

A risk averse agent is willing to accept a certainty-equivalent consumption plan which lies below the expectation of the stochastic consumption plan. That is, for any point in time t , we have $q_t \leq \mathbb{E}[\tilde{g}_t]$. q_t gets smaller, the comparatively more risk averse the agent is, i.e. the more the agent dislikes spread in consumption across states of nature, *ceteris paribus*. Similarly, holding risk preferences fixed, increasing risk in the sense of Rothschild and Stiglitz (1970) has comparable effects, i.e. q_t shrinks as \tilde{g}_t gets more risky, *ceteris paribus*.

The following key results naturally emerge from Equation 10 and the application of Jensen's inequality. First, holding all other things fixed, comparatively more risk averse agents will exhibit less pronounced changes in the certainty-equivalent consumption plan than others. This is the case as the evaluation of new alternatives involves consumption changes relative to the point in time the decision is made. Only today is certain, but future consumption is always subject to uncertainty. Risk aversion, however, lets q_t diminish, such that the effect on discount rates and the above discussed "anomalies" are comparatively less pronounced.

Second, increases in risk, by means of adding more noise to g_t , have similar effect. More uncertainty in the consumption plan means less pronounced declines in discount rates, a less pronounced magnitude effect and a less pronounced sign effect, *ceteris paribus*. These results contrast the findings in Epper and Fehr-Duda (2015b) who examine the risk of ending up with an outcome lower than the one promised (survival risk) and the risk attached to receiving the outcome itself (prospect risk). Survival risk together with Allais common ratio violations can generate hyperbolic-shaped discounting, the magnitude effect as well as many other important interaction effects between time and risk. Risk in outcomes can generate or amplify these effects, while risk in the consumption plan has the opposite (i.e. compressing) effect. The effect of anticipated changes in consumption are therefore very different from the expectation of potential income or wealth shocks. Accumulation of sufficient wealth, i.e. precautionary savings, can help the agent to overcome both, anticipated low-income periods and unanticipated fluctuations in income. Borrowing money against future labor income or the purchase of insurance are ways to resolve such issues, when no sufficiently large buffer-stock/liquid assets, but illiquid assets exists.

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