RISK IN TIME: The Intertwined Nature of Risk Taking and Time Discounting

Thomas Epper* Helga Fehr-Duda†

October 14, 2019

Abstract

Standard economic models view risk taking and time discounting as two independent dimensions of decision making. However, mounting experimental evidence demonstrates striking parallels in patterns of risk taking and time discounting behavior and systematic interaction effects, which suggests that there may be common underlying forces driving these interactions. Here we show that the inherent uncertainty associated with future prospects together with individuals' proneness to probability weighting generates a unifying framework for explaining a large number of puzzling behavioral regularities: delay-dependent risk tolerance, aversion to sequential resolution of uncertainty, preferences for the timing of the resolution of uncertainty, the differential discounting of risky and certain outcomes, hyperbolic discounting, subadditive discounting, and the order dependence of prospect valuation. Finally, we show that our framework also explains why people often simultaneously underinsure large risks and overinsure small risks.

JEL D01, D81, D91

*University of St. Gallen, School of Economics and Political Science, Varnbüelstrasse 19, 9000 St. Gallen, Switzerland; Center for Economic Behavior and Inequality (CEBI), University of Copenhagen, Øster Farimagsgade 5, 1353 Copenhagen, Denmark. Email: thomas.epper@unisg.ch.

†University of Zurich, Department of Banking and Finance, Plattenstrasse 32, 8032 Zurich, Switzerland. Email: helga.fehr@bf.uzh.ch.

1 Introduction

Whatever the nature of our decisions, hardly ever can we be sure about their outcomes. In particular, the consequences of the most important decisions in our lives, such as what line of business to enter or whom to get married to, do not materialize immediately but usually take quite some time to unfold. In other words, these important decisions involve both risk and delay. Driven by the evidence challenging Expected Utility Theory and Discounted Utility Theory, the past half century has seen a surge of new models of decision making for the domains of risk taking and time discounting (Starmer, 2000; Frederick, Loewenstein, and O'Donoghue, 2002; Wakker, 2010; Ericson and Laibson, 2019). A considerable body of evidence suggests, however, that risk taking and time discounting are linked and interact with each other in important ways (see Table 1 below).

First, risk aversion has been shown to be lower for risks materializing in the more remote future than for risks materializing in the more imminent future (e.g. Shelley (1994)).[1] Lower risk aversion for remote risks may be one reason why the mobilization of public support for policies combating global warming is so difficult. Thus, economic models of climate policy may benefit from recognizing that risk aversion decreases with time delay. Asset markets constitute another area where delay-dependent risk aversion may play an important role in understanding the downward sloping structure of risk premia, i.e. the fact that risk premia decline with maturity (van Binsbergen, Brandt, and Koijen, 2012).

A second fact is based on a considerable body of evidence that impatience tends to decrease when outcomes are shifted into the more remote future - a finding on which the large literature on hyperbolic discounting is based (Loewenstein and Thaler, 1989; Laibson, 1997). Third, the evidence indicates that risk taking is process dependent, i.e. the sequential evaluation of prospects over the course of time seems to render decision makers less risk tolerant (Bellemare, Krause, Kröger, and Zhang, 2005). In the domain of financial decisions, this phenomenon may underlie the large equity premia observed around the globe. Fourth, regarding time discounting, a similar phenomenon has been observed: discount rates compounded over subperiods tend to be higher than the discount rate applied to the total period. This incidence of process dependence, labeled *subadditive discounting*, has been put forward as an alternative explanation to hyperbolic discounting to account for the observed patterns in discounting behavior (Read, 2001; Dohmen, Falk, Huffman, and Sunde, 2017).

Fifth, many people also exhibit a preference regarding the timing of the resolution of uncertainty. This finding triggered a large theoretical literature following the seminal work of Kreps and Porteus (1978). A sixth regularity indicates that the presence of risk influences time discounting in an unexpected way: Sure outcomes are discounted more heavily than uncertain ones, discussed in the literature under the heading *diminishing immediacy* (Keren and Roelofsma, 1995; Weber and Chapman, 2005). Finally, people's evaluations of future risky payoffs depend

[1] In Section 6 we provide a detailed list of references regarding the empirical evidence and a discussion of extant theories that address various subsets of these facts.

on the order by which they are devalued for risk and for delay, which should not make any difference according to the standard view (Öncüler and Onay, 2009). These regularities suggest that theories that are restricted to either domain cannot easily account for the intertwined nature of risk taking and time discounting.

Table 1: Seven Facts on Risk Taking and Time Discounting

Dimension	Fact	Observed risk tolerance	Fact	Observed patience
Delay dependence	#1	increases with delay	#2	increases with delay
Process dependence	#3	higher for one-shot than for sequential valuation	#4	higher for one-shot than for sequential valuation
Timing dependence	#5	intrinsic preference for timing of uncertainty resolution		–
Risk dependence		–	#6	higher for risky payoffs than for certain ones
Order dependence	#7	depends on order of delay and risk discounting		–

The table describes seven regularities in experimental findings on risk taking and discounting behavior with respect to delay, process, timing, risk, and order effects.

A main purpose of our paper is to provide a unifying account of all these phenomena by integrating risk taking and time discounting into one theoretical approach. Thus, the goal is to develop a formal model that is capable of explaining all the regularities on the basis of a set of parsimonious assumptions. Our approach rests on two key assumptions: First, there is risk attached to any future prospect because only immediate consequences can be totally certain. We believe that this is a plausible assumption because it is impossible to foresee all future contingencies. Therefore, it is always possible that an event may occur that prevents the realization of a future outcome, i.e., something may go wrong before payoffs actually materialize. For example, an unforeseen contingency may arise, such as missing one's transatlantic flight because the taxi driver was late. Presumably, almost everyone can readily recall such an incident.

Second, if future prospects are perceived as inherently risky, people's risk tolerance must play a role in their valuations of future prospects. Therefore, the characteristics of (atemporal) risk preferences are crucial not only for evaluating delayed risky prospects but also for delayed (allegedly) certain ones. There is abundant evidence from the field and the laboratory that risk taking behavior depends nonlinearly on the objective probabilities (Prelec, 1998; Fehr-Duda and Epper, 2012; Barberis, 2013; O'Donogue and Somerville, 2018). For this reason, models involving probability weighting, such as Rank Dependent Utility Theory (RDU) (Quiggin, 1982) and Cumulative Prospect Theory (Tversky and Kahneman, 1992) have been strong contenders of Expected

Utility Theory (Wakker, 2010).

Our approach relies on a key characteristic of probability weighting, proneness to Allais-type *common-ratio violations*, that is one of the most widely replicated experimental regularities, found in human and animal behavior: Probabilistically mixing two lotteries with an inferior lottery frequently leads to preference reversals (Kahneman and Tversky, 1979; Gonzalez and Wu, 1999). This feature of probability weighting is called *subproportionality* and was characterized axiomatically in Prelec (1998).

Here we show for general n-outcome prospects that subproportional probability weighting in RDU together with the assumption that even (allegedly) certain future outcomes are inherently risky provides an integrative account of *all* the above mentioned empirical regularities. Thus, we rely on a well-established model of risk preferences with axiomatic foundations (e.g. Chateauneuf and Wakker (1999)) that we combine with the plausible assumption that something may go wrong in the future. In addition, we show that commonly used specifications of discount functions can be directly derived from our model of risk preferences. For example, the classical hyperbolic discount function, $\frac{1}{1+\gamma t}$ with $\gamma > 0$ (Mazur, 1987), is generated by the hyperbolic logarithm specification of probability weights, discussed in Prelec (1998).[2] Thus, subtle features of subproportionality, which distinguish different types of probability weighting functions, also imply different types of discount functions. Our model, therefore, creates new testable hypotheses concerning the link between risk taking and time discounting. Our results predict, in particular, that there should be distinct quantitative relationships between the parameters of estimated probability weights and discount functions.

Finally, we will show that our approach also provides a rationale for many real-world phenomena that have been puzzling economists for a long time. For example, the take-up of conventional life insurance contracts is notoriously low (Bernheim, Forni, Gokhale, and Kotlikoff, 2003; Cutler, Finkelstein, and McGarry, 2008), but many passengers were willing to pay exorbitant premiums for life insurance policies that covered the single flight they were about to take. The crucial feature of these different types of life insurance products is the timing of the resolution of uncertainty. Our model predicts that decision makers are much more risk averse when uncertainty resolves in the near future, as it does in the case of flight insurance, than when uncertainty is perceived to resolve in the more remote future, which arguably applies to regular life insurance.

While there is a large empirical and theoretical literature on the domain of risk taking and an equally large one on time discounting, there are, in comparison, relatively few papers dealing with an integrated view of risk and time (e.g. Prelec and Loewenstein (1991); Quiggin and Horowitz (1995); Baucells and Heukamp (2012)). Motivated by the similarities of anomalies in risk taking and time discounting behaviors, Prelec and Loewenstein (1991) develop psychological properties of multi-attribute prospect valuation that may be common in both decision domains.

[2] Another example is the flexible functional family of Constant Relative Decreasing Impatience discount functions (Bleichrodt, Rohde, and Wakker, 2009), which results from Prelec's famous Compound Invariance specification, applied in numerous experimental studies.

Thus, common ratio violations and decreasing impatience may be driven by the same psychological principles. The authors do not address how features of risk preferences and time preferences interact with each other, however. Similarly, Quiggin and Horowitz (1995) analyze parallels between the theories of choice under risk and choice over time and show the usefulness of Rank Dependent Utility Theory for understanding the analogy between risk aversion and impatience. Baucells and Heukamp (2012) link risk taking and time discounting by making direct assumptions on how people trade off delays in future outcomes against reductions in the probability with which these outcomes occur. They restrict their analysis to the case of simple prospects (x, p, t) that pay x with probability p at time t and zero otherwise. In this setting, the authors predict a number of effects by invoking varying additional assumptions.[3]

Halevy (2008), and later Saito (2011), demonstrate that there is a tight relationship between the concept of decreasing impatience and the crucial property of subproportionality, addressing Fact #2.[4] Our paper is inspired by this insight and extends it by showing that, beyond direct effects on time discounting, many additional important regularities in observed risk taking and time discounting follow from subproportional probability weighting and the assumption of an inherently uncertain future.

The remainder of the paper is organized as follows: Section 2 discusses key assumptions of our model. The predictions of our model are developed in Section 3. Section 4 is dedicated to the derivation of discount functions from various specifications of probability weighting functions, which establishes explicit links between the properties of probability weighting and discount functions. In Section 5 we discuss the implications of our findings for understanding other puzzling real-world behaviors. Section 6 relates our model in more detail to other theoretical approaches that address some of the regularities mentioned in Table 1. Finally, Section 7 concludes. Proofs and complementary materials are available in the appendix.

2 The Model

In the following, we will first present the general setup of our approach. Second, we justify our assumptions on the characteristics of the probability weighting function. Finally, we explain how we integrate that "something may go wrong" into the model.

2.1 Risk Preferences

In this paper, we rely on Rank Dependent Utility Theory (RDU), a generalization of Expected Utility Theory (EUT), that allows for nonlinear weighting of the probabilities. This additional

[3] Aside from its focus on simple prospects, their model assumes that prospects are played out and paid out at time t and, thus, does not speak to situations when uncertainty either resolves sequentially over time (Facts #3 and 4) or at some other time than the payoff date (Fact #5), nor does it address the order dependence of prospect valuation (Fact #7).

[4] Epper, Fehr-Duda, and Bruhin (2011) provide empirical support for Halevy's approach by showing experimentally that the degree of subproportionality of participants' probability weights predicts significantly and substantially the extent of their revealed decreasing impatience.

component has proven to be an exceptionally powerful force. First, overweighting of small probabilities may counteract risk aversion embodied in the utility function. Thus, RDU can handle the empirically observed probability dependence of risk tolerance (for an early example see Preston and Baratta (1948)). Probability weighting captures the intuition that "attention given to an outcome depends not only on the probability of the outcome but also on the favorability of the outcome in comparison to the other possible outcomes" (Diecidue and Wakker (2001), p. 284). Typically, decision makers focus on the worst and best possible outcomes and give much less attention to intermediate outcomes that will generally be underweighted even when they have the same objective probabilities as the extreme outcomes (Quiggin, 1982). Second, RDU displays first-order risk aversion, i.e. preferences between prospects whose consequences are sufficiently close to one another do not necessarily tend to risk neutrality. Thus, the experimental evidence of pronounced risk aversion over small stakes favors RDU over many other approaches that accommodate non-EUT behavior but display only second-order risk aversion (Sugden, 2004). Additionally, RDU was axiomatized by several authors and respects completeness, transitivity, continuity, and first-order stochastic dominance, qualities that many economists are hesitant to dispense with.[5]

According to RDU, a decision maker's atemporal risk preferences over prospects that are played out and paid out with negligible time delay can be represented by a rank-dependent functional. Consider a prospect $P = (x_1, p_1; ...; x_m, p_m)$ over (terminal) monetary outcomes $x_1 > x_2 > ... > x_m$ with $x_i \in X \subset \mathbb{R}, p_i \in [0,1]$ and $\Sigma p_i = 1$. The function u denotes the utility of monetary amounts x, and w denotes the subjective probability weight attached to p_1, the probability of the best outcome x_1. As usual, both u and w are assumed to be monotonically increasing, w to be twice differentiable and to satisfy $w(0) = 0$ and $w(1) = 1$. Decision weights π_i are defined as[6]

$$\pi_i = \begin{cases} w(p_1) & \text{for } i = 1 \\ w\left(\sum_{k=1}^{i} p_k\right) - w\left(\sum_{k=1}^{i-1} p_k\right) & \text{for } 1 < i \leq m \end{cases} \tag{1}$$

Thus, the decision weight of x_i is the probability weight attached to the probability of obtaining something at least as good as x_i minus the probability weight attached to the probability of obtaining something strictly better than x_i. Thus, decision weights sum to one. Finally, the prospect's value is represented by

$$V(P) = \sum_{i=1}^{m} u(x_i)\pi_i\,. \tag{2}$$

[5]RDU can handle correlation aversion as well: When decision makers evaluate risky consumption streams they often have a preference for diversifying consumption across time, i.e. they prefer some good and some bad to all or nothing (Kihlstrom and Mirman, 1974; Richard, 1975; Epstein and Tanny, 1980; Bommier, 2007; Denuit, Eeckhoudt, and Rey, 2010). Epper and Fehr-Duda (2015a) show that RDU implies correlation aversion if the decision maker is sufficiently pessimistic, which is usually borne out by the data.

[6]Alternatively, decision weights π_i can be expressed in terms of the cumulative distribution function F of the outcomes x_i: $\pi_i = w(1 - F(x_{i+1})) - w(1 - F(x_i))$ for $1 \leq i \leq m$, where $F(x_{m+1}) := 0$.

Rearranging terms in Equation 2 yields

$$\begin{aligned} V(P) &= u(x_1)w(p_1) + u(x_2)\Big(w(p_1+p_2) - w(p_1)\Big) + ... + u(x_m)\Big(1 - w(1-p_m)\Big) \\ &= \Big(u(x_1) - u(x_2)\Big)w(p_1) + ... + \Big(u(x_{m-1}) - u(x_m)\Big)w(1-p_m) + u(x_m)\,. \end{aligned} \tag{3}$$

This representation of V clarifies that x_m is effectively a sure thing whereas obtaining something better than x_m is risky.

If the prospect is not played out and paid out in the present, but at some future time $t > 0$, prospect value is affected by time discounting as well. We follow the standard approach and model people's willingness to postpone gratification by a constant rate of time preference $\eta \geq 0$, yielding a discount weight of $\rho(t) = \exp(-\eta t)$.[7] A prospect to be played out and paid out at $t > 0$ is discounted for time in the following standard way:

$$V_0(P) = V(P)\rho(t). \tag{4}$$

Abundant empirical evidence has demonstrated that risk taking behavior depends nonlinearly on the probabilities (Starmer, 2000; Fehr-Duda and Epper, 2012). However, in order to explain the observed interaction effects, we need to put more structure on the type of nonlinearity.

2.2 Probability Weighting

Inspired by one of Allais (1953)'s famous examples, Kahneman and Tversky (1979) presented subjects with the following experiment: Subjects had to choose between 3000 dollars for sure and 4000 dollars materializing with a probability of 80%. Most people chose the sure option of 3000 dollars. When confronted with the choice between a 25%-chance of receiving 3000 dollars and a 20%-chance of receiving 4000 dollars, the majority opted for the 4000-dollar alternative. Scaling down the probabilities of 100% and 80% by a common factor, in this example $1/4$, induced many people to reverse their preferences. Obviously, such Allais-type behavior is inconsistent with EUT.

An intuitive explanation for common-ratio violations is fear of disappointment: Losing a gamble over very likely 4000 dollars is anticipated to be much more disappointing than losing a gamble over 4000 dollars with only a small chance of materializing. On the other hand, winning 4000 dollars in the unlikely situation of a 20%-chance may trigger feelings of elation. Thus, when people are prone to disappointment and/or elation, their behavior appears to depend nonlinearly on the probabilities (Bell, 1985; Loomes and Sugden, 1986; Gul, 1991; Wu, 1999; Rottenstreich and Hsee, 2001; Walther, 2003).[8]

[7] This assumption is not crucial for our results - neither a zero rate of time preference, i.e. $\rho = 1$, nor genuinely hyperbolic time preferences affect our conclusions.

[8] Perceptual and procedural factors are potential drivers of probability distortions as well. The fathers of prospect theory, Kahneman and Tversky, attributed probability dependence to the psychophysics of perception according to which the sensitivity toward changes in probabilities diminishes with the distance to the natural reference points of certainty and impossibility (Tversky and Kahneman, 1992). Several other contributions focused on procedural

In RDU, common-ratio violations are mapped by subproportionality of probability weights. Formally, *subproportionality* holds if $1 \geq p > q > 0$ and $0 < \lambda < 1$ imply the inequality

$$\frac{w(p)}{w(q)} > \frac{w(\lambda p)}{w(\lambda q)} \tag{5}$$

(Prelec, 1998). Kahneman and Tversky (1979) note that this property imposes considerable constraints on the shape of w: it holds if and only if $\ln w$ *is a convex function of* $\ln p$. In other words, $\left(\frac{d \ln w}{d \ln p}\right)' > 0$, or the elasticity of w, $\varepsilon_w(p) = \frac{d \ln w}{d \ln p}$, is increasing in p. Define $\omega(\ln p) := \ln w(p)$. Then a subproportional w is equivalent to ω being a convex function of $\ln p$. Obviously, convexity can manifest itself in many different qualities - constant, increasing or decreasing absolute convexity; constant, increasing or decreasing relative convexity; a mix or even none of these. We will return to this issue in Section 4 when we discuss the implications of specific types of probability weighting functions for time discounting.

Subproportionality implies the *certainty effect*, which constitutes the special case of $p = 1$. Therefore,

$$w(\lambda q) > w(\lambda)w(q) \tag{6}$$

is satisfied for any λ, q such that $0 < \lambda, q < 1$.[9] This feature of subproportional probability weights has a crucial implication: It produces an aversion to compounding of probability weights (Segal, 1987a,b, 1990). We will use this insight when we discuss aspects of uncertainty resolution. There is also ample evidence for general common-ratio violations that do not involve a sure outcome (e.g. Loomes and Sugden (1987); Nebout and Dubois (2014)).

On average, estimated probability weighting curves overweight small probabilities and underweight large probabilities of the best outcome, which is also a common characteristic of individual estimates (Gonzalez and Wu, 1999; Bruhin, Fehr-Duda, and Epper, 2010). This feature is labeled regressiveness:[10]

A probability weighting function $w(p)$ is *regressive* if there exists a probability $p^* \in (0,1)$, such that

$$\begin{array}{ll} w(p) > p & \text{for } p < p^* \\ w(p) = p^* & \text{for } p = p^* \\ w(p) < p & \text{for } p > p^* \end{array} . \tag{7}$$

aspects of choice (Rubinstein, 1988; Loomes, 2010). In these models, a prospect's value depends not only on the prospect's own characteristics but also on other prospects in the choice set. A recent contribution in this category is Bordalo, Gennaioli, and Shleifer (2012) who posit that probabilities are distorted in favor of payoffs that are perceived as particularly salient. Whatever its cause, we interpret probability weighting in rank-dependent models as a kind of reduced form generated by some psychological mechanism.

[9]The numerical example above is a manifestation of the certainty effect, as the smaller outcome in the first decision situation, 3000 dollars, materializes with certainty.

[10]Aside from regressive shapes, convex weighting curves which globally underweight probabilities comprise another common category of individuals' probability weighting functions (see e.g. van de Kuilen and Wakker (2011)), which may be subproportional as well. Empirical estimates are often based on inverse-S shaped functional forms, that are concave over small probabilities and convex over large probabilities. For our purposes, the less demanding assumption of regressiveness is sufficient.

In the context of rank-dependent models, regressiveness of the probability weighting function generates overweighting of a prospect's tail outcomes and underweighting of its intermediate outcomes, which nicely captures the notion that more extreme outcomes within a given prospect are more salient. While the driver of our results is subproportionality, regressiveness is an independent additional characteristic that captures key features of real-world behavior, discussed in Section 5.[11]

Many functional specifications proposed in the literature exhibit subproportionality over some probability range under appropriate parameter restrictions (see Appendix B.2). Perhaps the most prominent representative of a globally subproportional function is Prelec (1998)'s flexible two-parameter specification, designed to map common-ratio violations. Throughout the paper, we will use this "standard" functional specification, which is also regressive for typical parameter estimates, to illustrate our results.

2.3 Future Uncertainty

The final building block of our model concerns the integration of "something may go wrong" due to unexpected contingencies. This (uninsurable) risk inherent in the future, *survival risk* for short, turns allegedly guaranteed payoffs into risky ones and introduces an additional layer of risk over and above the objective probability distributions of risky payoffs (henceforth referred to as *prospect risk*). Consequently, there are two distinct types of risk, *prospect risk* which may resolve at any time between the present and the payoff date, and *survival risk* which resolves fully only at the payoff date. Thus, the subjective perception of future uncertainty changes the nature of the prospect. Formally, let $0 < s < 1$ denote the constant per-period probability of prospect survival, i.e. the probability that the decision maker will actually obtain the promised rewards by the end of the period. Essentially, there are two ways of accounting for this subjective probability s. First, for a delay t, the probability s^t is transformed according to the decision maker's probability weighting function, and the resulting $w(s^t)$ affects the prospect as a whole, i.e. all outcomes equally. In this case, prospect value amounts to

$$V_0(P) = V(P)w(s^t)\rho(t). \tag{8}$$

Such an approach only affects measured discount rates but cannot handle the observed interaction effects. Thus, we work with the second solution, namely that s impacts the probability distribution of the prospect. Then the probability that the allegedly guaranteed payment x_m materializes at the end of period t is perceived to be s^t, and the probabilities of obtaining something better than x_m are scaled down by s^t. Therefore, the objective m-outcome prospect is subjec-

[11] A recent experimental paper on risky delayed prospects contests the usefulness of probability weighting models, however: Andreoni and Sprenger (2012) claim that their findings on intertemporal risk aversion cannot be explained by probability weighting. Contrary to their claim, Epper and Fehr-Duda (2015b) demonstrate that probability weighting in RDU not only accommodates intertemporal risk aversion but also provides, among several candidate models, the most convincing account of their data.

tively perceived as an *(m+1)*-outcome prospect $\tilde{P} = \left(x_1, p_1 s^t; x_2, p_2 s^t; ...; x_m, p_m s^t; \underline{x}, 1 - s^t\right)$, where $\underline{x}$ captures that "something may go wrong".

Setting $u(\underline{x}) = 0$, the subjective present value of the prospect amounts to

$$\begin{aligned}
[V(\tilde{P})]_0 = & \Big(\Big(u(x_1) - u(x_2)\Big)w(p_1 s^t) + ... \\
& ... + \Big(u(x_{m-1}) - u(x_m)\Big)w\Big((1 - p_m)s^t\Big) + u(x_m)w(s^t)\Big)\rho(t) \\
= & \Big(\Big(u(x_1) - u(x_2)\Big)\tfrac{w(p_1 s^t)}{w(s^t)} + ... \\
& ... + \Big(u(x_{m-1}) - u(x_m)\Big)\tfrac{w\left((1-p_m)s^t\right)}{w(s^t)} + u(x_m)\Big)w(s^t)\rho(t)\,.
\end{aligned} \tag{9}$$

From the point of view of an outsider observer, the subjective probability distribution of prospect P is not observable. Consequently, she infers probability weights $\tilde{w}$ and discount weights $\tilde{\rho}$ from observed behavior on the presumption that the decision maker evaluates the objectively given prospect P, and estimates preference parameters according to RDU in the standard way:

$$[V(\tilde{P})]_0 = \Big(\Big(u(x_1) - u(x_2)\Big)\tilde{w}(p_1) + ... + \Big(u(x_{m-1}) - u(x_m)\Big)\tilde{w}(1 - p_m) + u(x_m)\Big)\tilde{\rho}(t)\,, \tag{10}$$

interpreting $\tilde{w}$ as true probability weights and $\tilde{\rho}$ as true discount weights, while in fact the weights are distorted by survival risk. By comparing Equation 9 with Equation 10 we can see that the relationships between true and observed weights are given by

$$\tilde{w}(p) = \frac{w(ps^t)}{w(s^t)}, \tag{11}$$

$$\tilde{\rho}(t) = w(s^t)\rho(t). \tag{12}$$

These equations define the central relationships between observed and true underlying probability and discount weights. Concerning observed discount weights, a representation equivalent to Equation 12 was derived by Halevy (2008) in the context of Yaari (1987)'s dual theory. Note that Equation 12 also holds for observed discount weights based on the separable model of Equation 8. Thus, if discounting of dated sure outcomes is the sole focus of research, it does not matter in which way the probability of prospect survival is integrated into the model. Therefore, it takes at least two non-zero outcomes to be able to separate risk taking from time discounting. As will become clear below, the choice of integration method is crucial for explaining delay dependent risk tolerance and preferences for resolution timing.

Since $\tilde{w}(p) \neq w(p)$ and $\tilde{\rho}(t) \neq \rho(t)$ for subproportional preferences, survival risk drives a wedge between true underlying preferences and observed risk taking and discounting behavior. Thus, future risk conjointly with proneness to Allais-type behavior provides the mechanism by which behavior under risk and behavior over time are intertwined. A summary of the model variables is provided in Table 2.

Table 2: Model Variables

	Variable	Description	Characteristics
Prospects	x	monetary payoff	$x \geq 0$
	p	probability of x	$0 \leq p \leq 1$
	s	probability of prospect survival	$0 < s < 1$
	$1-s$	survival risk	
	t	length of time delay	$t \geq 0$
Preferences	$u(x)$	utility function	$u(0) = 0, u' > 0$
	$w(p)$	true probability weight	$w(0) = 0, w(1) = 1, w' > 0$
	η	rate of pure time preference	$\eta \geq 0$, constant
	$\rho(t)$	discount weight	$\rho(t) = \exp(-\eta t)$
Behavior	$\tilde{w}(p)$	observed probability weight	$\tilde{w}(p) = w(ps^t)/w(s^t)$
	$\tilde{\rho}(t)$	observed discount weight	$\tilde{\rho}(t) = w(s^t)\rho(t)$
	$\tilde{\eta}(t)$	observed discount rate	$\tilde{\eta}(t) = -\tilde{\rho}'(t)/\tilde{\rho}(t)$

3 Unifying the Experimental Evidence

In the following, we discuss the implications of our approach for the experimental phenomena listed in Table 1 and demonstrate that all the Facts #1 through #7 can be explained within our framework. An important feature of future prospects concerns the timing of the resolution of uncertainty. We distinguish three different cases: First, the prospect is played out and paid out at the same time, i.e. both prospect risk and survival risk are resolved simultaneously in one shot at the payment date t. This situation refers to the issue of delay dependence. Second, both prospect and survival risk are resolved sequentially over the course of time, which is the topic of process dependence. Finally, prospect risk is resolved before the payment date, setting the focus on resolution timing. The proofs of the respective propositions are presented in Appendix A.

3.1 Facts #1 and #2: Delay Dependence

In the following, prospect risk and survival risk are resolved simultaneously in one shot at time t. Starting with the analysis of risk tolerance, we see from Equation 11 that observed probability weights $\tilde{w}(p)$ deviate systematically from the underlying atemporal ones $w(p)$.

PROPOSITION 1:

Given subproportionality of $w, t > 0$ and $s < 1$:

1. The function $\tilde{w}$ is a proper probability weighting function, i.e. monotonically increasing in p with $\tilde{w}(0) = 0, \tilde{w}(1) = 1$.

2. $\tilde{w}$ is subproportional.

3. $\tilde{w}$ is more elevated than w: $\tilde{w}(p) > w(p)$. Elevation increases with
   - time delay t,
   - survival risk $1 - s$, and
   - comparatively more subproportional w.

4. Relative elevation $\frac{\tilde{w}(p)}{w(p)}$ declines in p.

5. $\tilde{w}$ is less elastic than w.

6. The decision weight of the (objectively) worst possible outcome, x_m, decreases with delay t.

[Proof in Appendix A]

That $\tilde{w}$ is more elevated than w constitutes one of the central implications of our model.[12] It follows directly from subproportionality:

$$\tilde{w}(p) = \frac{w(ps^t)}{w(s^t)} > \frac{w(p)}{w(1)} = w(p). \tag{13}$$

Since the probability weighting function maps the decision weight of the best possible outcome, an increase in the elevation of the probability weighting curve gets directly translated into higher revealed risk tolerance.[13] Thus, the presence of survival risk makes people appear more risk tolerant for delayed prospects than for present ones. Intuitively, the event of something going wrong takes on the role of the perceived sure outcome, which makes x_m an intermediate one and, thus, less salient to the decision maker. In addition, this risk-tolerance increasing effect is particularly strong for small probabilities, i.e. positively skewed prospects are subject to more pronounced increases in risk tolerance.

[12] Clearly, subproportionality is sufficient to produce all the results of Propositions 1 (as well as of the following propositions). But is subproportionality, aside from survival risk $1 - s > 0$, also necessary? For statements that refer to the present it is necessary that preferences exhibit the certainty effect, i.e. that $w(p)w(q) < w(pq)$ for any $p, q < 1$, which is implied by but does not imply subproportionality. Therefore, the result that risk tolerance is higher for future prospects than for present ones does not depend on subproportionality, only on the certainty effect. However, for statements pertaining to relationships between behaviors at different times in the future, for instance, that risk tolerance is increasing in t or that discount weights decline hyperbolically, subproportionality is necessary (for a proof with respect to observed discount weights see Saito (2011)). For example, preferences that are not generally subproportional but exhibit the certainty effect, such as the discontinuous weighting function $w(p) = \gamma p$ for $p < 1$ and $w(1) = 1$ defined for $0 < \gamma < 1$, will show an increase in risk tolerance relative to the present as well as quasi-hyperbolic discounting.

[13] In the domain of simple prospects (x, p), Baucells and Heukamp (2012) derive a time-dependent probability weighting function $\tilde{w}(p) = w(p \exp(-r_x t))$, which obviously decreases with t. A crucial element of their model is r_x, the probability discount rate that is assumed to decrease with outcome magnitude. Together with the common ratio effect, this additional assumption drives their result that risk premia decline with time delay.

The delay dependence of observed probability weights $\tilde{w}$ is illustrated in Figure 1. The top row of Figure 1 characterizes preferences in the atemporal case. Panel 1a shows a typical specimen of a regressive probability weighting function w for delay $t = 0$, underweighting large probabilities and overweighting small probabilities of the best outcome. For illustrative purposes, Panel 1b on the right side depicts the corresponding decision weights π_i for a prospect involving 21 equiprobable outcome levels, with outcome rank 1 denoting the best outcome and outcome rank 21 the worst one. Their objective probabilities are represented on the horizontal gray line. As one can see, a regressive w generates strong overweighting of the extreme outcomes and underweighting of the intermediate ones relative to the objective probability distribution.

The middle row of Figure 1 demonstrates the predictions for one-shot resolution of uncertainty, i.e. when prospects are played out and paid out simultaneously in the future, the focus of Proposition 1. Future uncertainty is captured by the parameter $s = 0.8$, i.e. the per-period prospect survival rate is perceived to be 80%. When payoffs are delayed by two periods, $t = 2$, and uncertainty resolves in one shot ($n = 1$), observed probability weights $\tilde{w}$ shift upwards, as shown in Panel 2a. This shift rotates the decision weights $\tilde{\pi}_i$ counterclockwise, as depicted in Panel 2b. Now the worst outcomes are underweighted while the best ones are more strongly overweighted. For longer time delays these effects become more pronounced and may lead to a substantial underweighting of the worst outcomes. Thus, underweighting of adverse extreme events becomes more likely with longer time horizons. Note that, due to Proposition 1.5, the weight of the (objectively) worst outcome x_m will decline with t, whatever the shape of the subproportional function.

Allegedly guaranteed future payoffs constitute a special case of risky ones. As evident in Equation 12, the observed discount weight for time equals $\tilde{\rho}(t) = w(s^t)\rho(t)$. Clearly, if w is linear, $\tilde{\rho}$ declines exponentially irrespective of the magnitude of s. To see this, note that $\rho(t) = \exp(-\eta t)$ and $s^t = \exp\Big(- \big(- \ln(s)\big) t\Big)$, implying a discount rate $\tilde{\eta} = \eta - \ln(s) > \eta$ for $0 < s < 1$. In this case, uncertainty *per se* increases the absolute level of revealed impatience, but cannot account for declining discount rates. Thus, an expected-utility maximizer will exhibit a constant discount rate that is higher than her underlying rate of pure time preference, but her behavior will not show any of the interaction effects addressed in this paper. If, however, w is subproportional and $0 < s < 1$, the component $w(s^t)$ distorts the discount weight in a predictable way, set out in Proposition 2. Prediction 2.3 is essentially equivalent to Theorem 1 in Halevy (2008) applied to our framework.

PROPOSITION 2

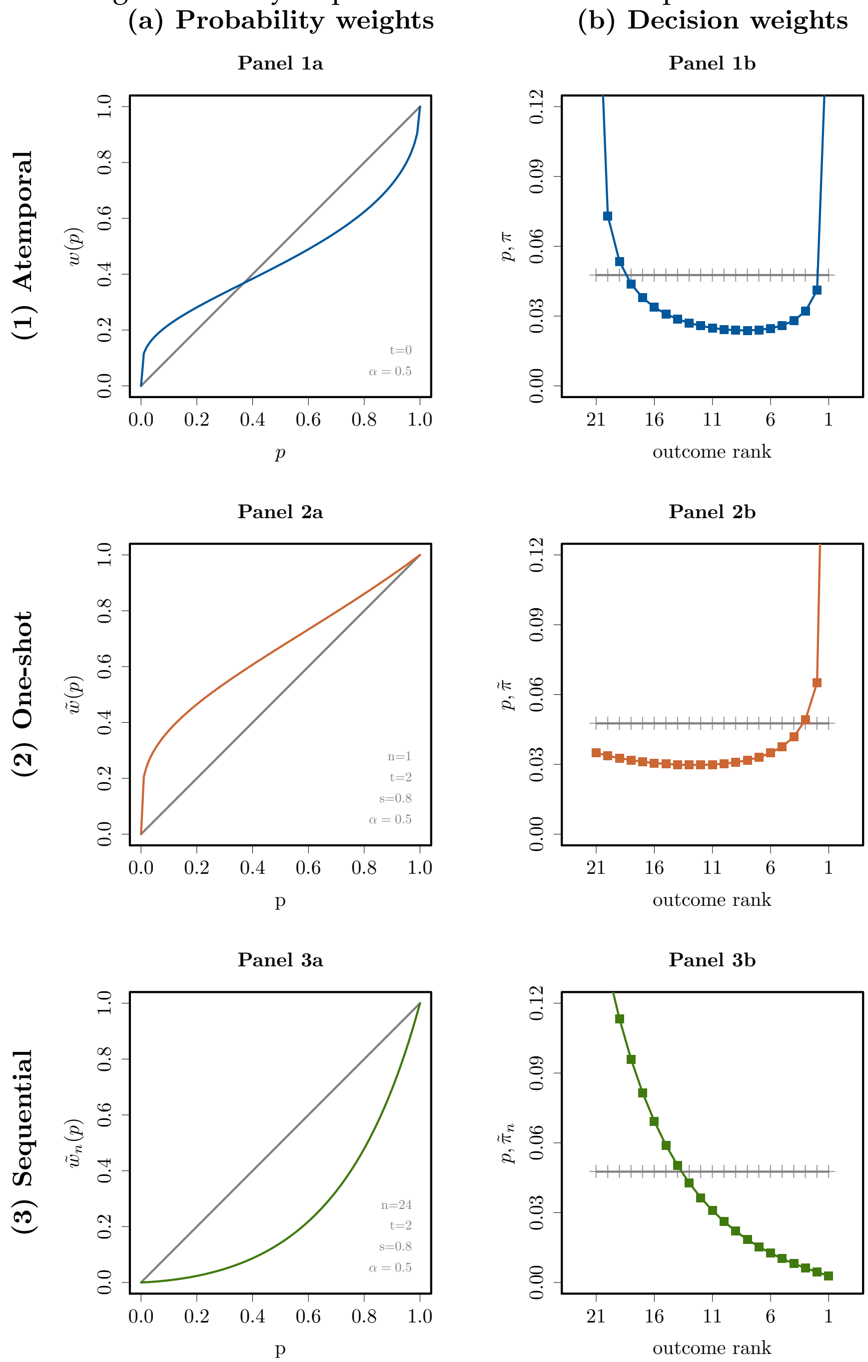
Given subproportionality of w:

1. $\tilde{\rho}(t)$ is a proper discount function for $0 < s \leq 1$, i.e. decreasing in t, converging to zero with $t \to \infty$, and $\tilde{\rho}(0) = 1$.

2. Observed discount rates $\tilde{\eta}(t)$ are higher than the rate of pure time preference η for $s < 1$.

3. Observed discount rates decline with the length of delay for $s < 1$.

Figure 1: Delay Dependence and Process Dependence

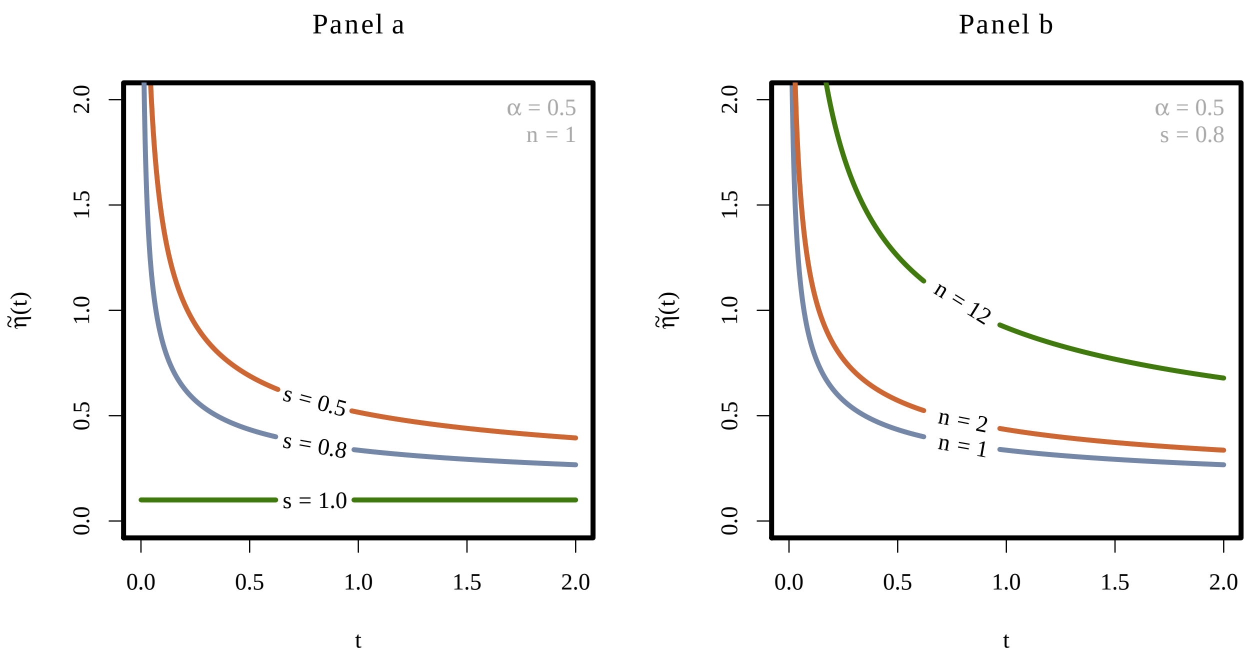


For purposes of illustration, the curves are derived from Prelec (1998)'s two-parameter probability weighting function $w(p) = \exp\big(-\beta(-\ln(p))^{\alpha}\big)$, assuming a degree of subproportionality $\alpha = 0.5$ and convexity $\beta = 1$. Survival risk s is set at 0.8 per period. n denotes the number of (equally spaced) stages in the case of sequential evaluation. **Top row (1. atemporal)**: The graphs show atemporal probability weights w (Panel 1a) and their associated decision weights π (Panel 1b) for a prospect involving 21 equiprobable outcomes, with outcome rank 1 denoting the best outcome. Their objective probabilities are represented on the horizontal gray line. **Middle row (2. one-shot)**: Panel 2a and 2b show $\tilde{w}$ and $\tilde{\pi}$ for a delay of two periods, $t = 2$, when uncertainty resolves in one shot $n = 1$. **Bottom row (3. sequential)**: Panel 3a and 3b show $\tilde{w}$ and $\tilde{\pi}$, respectively, for a delay of two periods when uncertainty resolves sequentially in $n = 24$ equally spaced stages, $\tilde{w}(p) = \left(\frac{w\left((ps^t)^{1/n}\right)}{w\left((s^t)^{1/n}\right)}\right)^n$.

4. Greater survival risk generates a greater departure from constant discounting.
5. Comparatively more subproportional probability weighting generates a comparatively greater departure from constant discounting.

[Proof in Appendix A]

Figure 2: Hyperbolic and Subadditive Discount Rates $\tilde{\eta}$



Panel a shows discount rates as they move with the length of delay t for different levels of survival risk $1-s$, where s denotes the probability of prospect survival. When there is no survival risk, $s = 1$, the observed discount rate is constant and equals the rate of pure time preference (line labeled by $s = 1.0$). The higher is the level of risk, the lower s, the more pronounced the hyperbolic decline of discount rates over time is for decision makers with subproportional probability weights (curves labeled by $s = 0.5$ and $s = 0.8$). $\tilde{\eta}(t) := -\frac{\partial \tilde{\rho}}{\partial t} / \tilde{\rho}$. w is specified as Prelec's probability weighting function (in this example $\alpha = 0.5$ and $\beta = 1$). **Panel b** depicts discount rates for a constant level of survival probability $s = 0.8$ and varying number of resolution stages n. The more often a particular delay is divided into subintervals (of equal length in this graph), the higher is the discount rate, a manifestation of subadditive discounting.

In our model, decreasing impatience is not a manifestation of pure time preferences but a consequence of survival risk changing the subjective nature of future prospects. At the level of observed behavior, decreasing impatience is the mirror image of increasing risk tolerance if survival risk is integrated into the prospect's probability distribution. In fact, the degree of proneness to common-ratio violations, the degree of subproportionality, can be interpreted as degree of time insensitivity. Intuitively, when the future is inherently risky, promised rewards do not materialize with certainty and, therefore, they incorporate the potential of disappointment. Because more immediate payoffs are more likely to actually materialize than more remote payoffs, this potential is perceived to decline with the passage of time and becomes almost negligible for payoffs far out in the future. Technically, since shifting a payoff into the future amounts to scaling down its probability, a decision maker with subproportional preferences becomes progressively insensitive to a given timing difference.[14] This insight provides a testbed for analyzing risk taking and time discounting behavior at the individual level because the characteristics of the probability weighting function feed directly into the characteristics of the discount function.

Interestingly, the most widely used specifications of discount functions that can accommodate decreasing impatience, the hyperbolic discount function, the Constant Relative Decreasing Impatience (CRDI) and the Constant Absolute Decreasing Impatience (CADI) specifications can be directly derived from specific varieties of subproportional probability weighting functions, as we show in Section 4.

The effects of survival risk on revealed discount rates are presented in Panel a of Figure 2, which depicts a typical decision maker's observed *discount rates* $\tilde{\eta}$ as they react to varying levels of s. The horizontal line represents the case of no survival risk, $s = 1$. In this case, the observed discount rate $\tilde{\eta}$ is constant and coincides with the true underlying rate of time preference η. When survival risk comes into play, however, discount rates decline in a hyperbolic fashion, and depart from constant discounting increasingly strongly with rising uncertainty, as shown by the curves for $s = 0.8$ and $s = 0.5$, respectively.

3.2 Facts #3 and #4: Process Dependence

So far, we have considered the case of uncertainty resolving in one shot. In this Section we deal with uncertainty resolving sequentially over the course of time. Specifically, we examine the case when both prospect risk and survival risk are resolved simultaneously over the course of time. If uncertainty resolves sequentially, future prospects turn into multi-stage ones. In this case, the question arises in which way multi-stage prospects are transformed into single-stage ones, the domain over which risk preferences are defined. In principle, there are two different transformation methods, reduction by probability calculus and folding back. Reduction involves the calculation of the probabilities of the final outcomes and the transformation of these values by the appropriate weighting function. Folding back, on the other hand, weights the probabilities at each stage and then compounds these weights. It is well known that a naive RDU decision maker will be dynamically inconsistent if she cares only about the probabilities of the final outcomes - as the payment date draws near, she will re-evaluate the prospect and, because of the delay dependence of risk tolerance, become comparatively more risk averse.

Folding back, on the other hand, ensures dynamic consistency but has substantial consequences for revealed risk taking behavior. As already noted, the certainty effect embodied in subproportional preferences generates an aversion to compounded probability weights: For $1 > p = q_1 q_2 > 0$ the compounding of the respective weights always leads to lower prospect

[14] Saito (2015) shows that in the continuous case for $\rho = 1$ the common ratio effect and hyperbolic discounting are equivalent. $\rho = 1$ indicates that the only source of time preference is future uncertainty.

values, i.e. $w(p) > w(q_1)w(q_2)$ holds whatever are the values of q_1 and q_2. But can we say more than that? What happens when the number of stages increases from 2 to $n > 2$ and in which way does the specific composition of the probability p affect prospect value?

In the following, we set $\rho = 1$ for ease of exposition. Let us first consider a two-outcome prospect $P = (x_1, p; x_2)$ resolving in two stages, $n = 2$, denoted by corresponding subscripts to $\tilde{w}$ and $\tilde{\rho}$, such that uncertainty is partially resolved at some future time t_1 and fully resolved at the payment date $t > t_1$, as depicted in Figure 3. Applying folding back, the resulting two-stage prospect is evaluated as

$$\begin{aligned}[V_2(\tilde{P})]_0 &= \Big(u(x_1) - u(x_2)\Big) w\left(p^{\frac{t_1}{t}} s^{t_1}\right) w\left(p^{\frac{t-t_1}{t}} s^{t-t_1}\right) + u(x_2) w\left(s^{t_1}\right) w\left(s^{t-t_1}\right) \\ &= \left(\Big(u(x_1) - u(x_2)\Big) \frac{w\left(p^{\frac{t_1}{t}} s^{t_1}\right) w\left(p^{\frac{t-t_1}{t}} s^{t-t_1}\right)}{w\left(s^{t_1}\right) w\left(s^{t-t_1}\right)} + u(x_2)\right) w\left(s^{t_1}\right) w\left(s^{t-t_1}\right) \\ &= \left(\Big(u(x_1) - u(x_2)\Big) \tilde{w}_2(p) + u(x_2)\right) \tilde{\rho}_2(t)\,,\end{aligned} \tag{14}$$

which yields the relationships

$$\tilde{w}_2(p) = \frac{w\left(p^{\frac{t_1}{t}} s^{t_1}\right) w\left(p^{\frac{t-t_1}{t}} s^{t-t_1}\right)}{w\left(s^{t_1}\right) w\left(s^{t-t_1}\right)}\,, \tag{15}$$

$$\tilde{\rho}_2(t) = w\left(s^{t_1}\right) w\left(s^{t-t_1}\right)\,, \tag{16}$$

as $\tilde{\rho}_2(t)$ is interpreted as the discount weight attached to the allegedly certain outcome x_2. Subproportionality ensures that

$$\tilde{w}_2(p) = \frac{w\left(p^{\frac{t_1}{t}} s^{t_1}\right) w\left(p^{\frac{t-t_1}{t}} s^{t-t_1}\right)}{w\left(s^{t_1}\right) w\left(s^{t-t_1}\right)} < \frac{w(ps^t)}{w(s^t)} = \tilde{w}(p)\,, \tag{17}$$

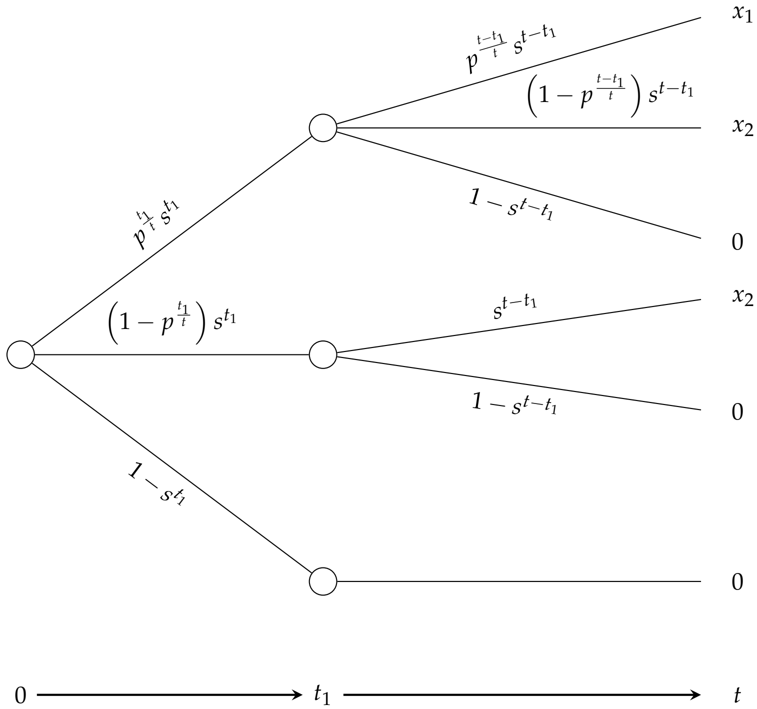
one of the main results generalized in Proposition 4. Now suppose that the interval $[0, t]$ is partitioned into n subintervals with lengths τ_i, $i \in \{1, ..., n\}$, such that $\sum_{i=1}^{n} \tau_i = t$. In this case, it is straightforward to show that, for any number of outcomes $m \geq 1$, the observed probability weights are given by

$$\tilde{w}_n(p, t) = \frac{\prod_{i=1}^{n} w\left(p^{\frac{\tau_i}{t}} s^{\tau_i}\right)}{\prod_{i=1}^{n} w\left(s^{\tau_i}\right)} = \prod_{i=1}^{n} \tilde{w}\left(p^{\frac{\tau_i}{t}}, \tau_i\right)\,. \tag{18}$$

The following Proposition 3 summarizes our insights on subproportional probability weights w themselves, which drive overall prospect value, without teasing apart the separate effects on observed risk tolerance and discounting behavior. We extend these results to observed risk tolerance $\tilde{w}$ in Proposition 4. Since discount weights $\tilde{\rho}(t) = w(s^t)$ are simple probability weights themselves, Proposition 3 also speaks directly to observed discounting behavior.

PROPOSITION 3:

Figure 3: Sequential Resolution of Uncertainty



Given subproportionality of w, $s < 1$, $t > 0$, prospect risk and survival risk resolving simultaneously, and folding back:

1. For any number of resolution stages $n > 1$, probability weights w for one-shot resolution of uncertainty are greater than compounded probability weights for sequential resolution.

2. For a given number of resolution stages n, probability weights are smallest for evenly spaced partitions $\tau_i = \frac{t}{n} = \tau$.

3. For evenly spaced partitions, probability weights decline with the number of resolution stages n.

[Proof in Appendix A]

Proposition 3.1 generalizes the implications of the certainty effect to any number of stages. Thus, total prospect value is always smaller for sequential resolution of uncertainty. In other words, in our setting a decision maker exhibits a preference for one-shot resolution of uncertainty (Fact #3) as well as subadditive discounting (Fact #4). However, a preference for one-shot resolution of uncertainty does not hold generally under subproportionality in RDU but only applies to the class of prospects studied here, i.e. prospects that are devalued by survival risk without effects on the rank order of the outcomes. For details see our discussion in Appendix B.1.

We also find that a prospect's minimum value is attained when compounding occurs over equiprobable stages. This finding constitutes a generalization of Segal (1990)'s result for atemporal two-stage lotteries. Partitions of equal length correspond to the least degenerate multistage prospect and can be interpreted as the comparatively most ambiguous situation, which is strongly disliked by people with subproportional preferences.[15] Finally, we find a clear ranking of prospect values in the case of partitions of equal length, namely, increasing the number of equiprobable stages depresses prospect value.

Proposition 3 sets out the implications of subproportionality on overall prospect value. Thus, in the case of time discounting, we predict both decreasing impatience *and* subadditivity as well as an aversion to discounting over evenly spaced subintervals. Panel b of Figure 2 shows the effect of varying the number of compounding stages on observed discount rates. As predicted, discount rates increase in the number of stages. In our model, subadditive discounting is the result of decision makers' aversion to compounded probability weights and not a feature of pure time preferences themselves, as posited in the literature.

Since observed risk tolerance depends on the interaction of probability weights and discount weights (subproportional probability weights themselves), it is a priori not clear whether all these characteristics carry over to observed risk tolerance. As it turns out, with one exception, the characteristics of subproportional probability weights shape observed delay-dependent risk tolerance accordingly:

PROPOSITION 4:

Given subproportionality of w, $s < 1$, $t > 0$, prospect risk and survival risk resolving simultaneously, and folding back:

1. For any number of resolution stages $n > 1$, risk tolerance is higher for one-shot resolution of uncertainty than for sequential resolution of uncertainty, $\tilde{w}(p,t) > \tilde{w}_n(p,t)$.
2. For a given number of resolution stages n, risk tolerance is lowest for evenly spaced partitions if the elasticity of w is concave.
3. For evenly spaced partitions, risk tolerance declines with the number of resolution stages, $\tilde{w}_n(p,t) < \tilde{w}_{n-1}(p,t)$.

[Proof in Appendix A]

As we have shown in Proposition 1.2, $\tilde{w}$ is subproportional itself and, therefore, Proposition 4.1 does not come as a big surprise. If prospect risk and survival risk are resolved simultaneously in one shot, not only overall prospect value but also the separate components $\tilde{w}$ and $\tilde{q}$ attain their maximum values. If uncertainty resolves sequentially, both types of weights are smaller than

[15] Because of this characteristic, Segal (1987b) proposes to model ambiguity aversion by subproportional risk preferences over two-stage lotteries. A recent paper by Dillenberger and Segal (2014) shows that such an approach has another attractive implication: It is able to solve Machina (2009, 2014)'s paradoxes which involve a number of situations where standard models of ambiguity aversion are unable to capture plausible features of ambiguity attitudes (Baillon, l'Haridon, and Placido, 2011).

in the one-shot case. For evenly spaced partitions, this effect gets more pronounced the finer is the partition of delay t into subintervals. Therefore, anticipating to watch uncertainty resolve over time considerably dampens the effect of long time horizons on observed risk tolerance because, intuitively, the decision maker is frequently exposed to the possibility of a disappointing outcome.

Contrary to the underlying probability weights w themselves, subproportionality alone does not guarantee that, for a given number of resolution stages, risk tolerance $\tilde{w}$ attains its minimum at evenly spaced partitions. The additional requirement of concavity of the elasticity of w implies that the elasticity increases more quickly for small probabilities than for large ones. While such a characteristic has not attracted any attention in the literature, there is a nice specimen of a subproportional regressive probability weighting function with concave elasticity, the so-called *neo-additive* specification

$$w(p) = \begin{cases} 0 & \text{for } p = 0 \\ \beta + \alpha p & \text{for } 0 < p < 1 \\ 1 & \text{for } p = 1 \end{cases} . \tag{19}$$

with $0 < \beta < 1, 0 < \alpha \leq 1 - \beta$. If $\beta = 0$, w is not subproportional, for $\alpha + \beta = 1$ it is not regressive. It is linear over the inner probability interval and, thus, provides an excellent approximation for the commonly used nonlinear functional forms. Since we rarely, if at all, have experimental evidence for behavior over probabilities that are extremely small or extremely large, such an approximation seems justified. This specification is also very useful for the case of ambiguity, when the probabilities are not precisely known (Chateauneuf, Eichberger, and Grant, 2007).

The bottom row of Figure 1 demonstrates the effect of sequential valuation on observed probability weights $\tilde{w}$ and their corresponding decision weights for a delay of $t = 2$. If a prospect is evaluated in 24 evenly spaced time intervals, $n = 24$, the probability weighting curve takes on a convex form, which implies strong risk aversion. The associated decision weights for our reference prospect involving 21 equiprobable outcomes are depicted in Panel 3b. The decision weight curve now rotates clockwise: The worst outcomes are strongly overweighted while the best outcomes are considerably underweighted. Sequential valuation, therefore, has a dramatic effect on the overweighting of adverse tail events. This effect may be called *myopic probability weighting* in the style of myopic loss aversion (Benartzi and Thaler, 1995) which has similar consequences on risk taking behavior when short-sighted investors are frequently exposed to the possibility of facing losses.

3.3 Fact #5: Timing Dependence of Risk Tolerance

The previous theoretical result rests on the assumption that prospect risk is resolved simultaneously with survival risk. If the prospect is played out before payment takes place, prospect risk is segregated from survival risk. As long as prospect risk is unresolved, both types of risks are effective, after resolution of prospect risk only survival risk remains to be resolved, defining two distinct stages of uncertainty resolution. As Segal (1990) argues, folding back is particularly plausible when the stages are clearly distinct.

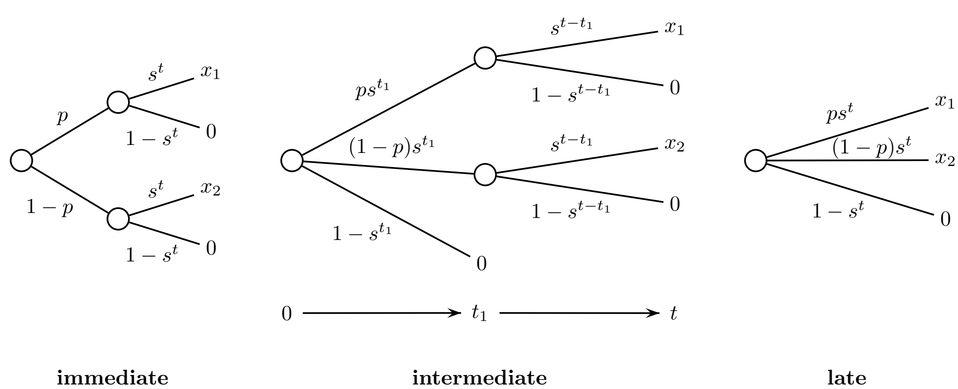
Figure 4 depicts three different cases: First, the prospect is played out immediately after prospect valuation. In this case, the decision maker will know the outcome right after her decision and faces only survival risk. This situation corresponds to the left panel, labeled "immediate". In fact, this case corresponds to the representation in Equation 8 where survival risk affects prospect value as a whole and not its probability distribution. The right panel shows the other extreme, labeled "late" when the prospect is played out and paid out at the same time t, handled by Propositions 1 and 2. The middle panel is dedicated to the intermediate case when prospect risk is resolved at some time t_1 in the future before the final payment date t, the focus of Proposition 4.

The corresponding formulas for the respective probability and discount weights are given by

$$\tilde{w}(p;t,t_1) = \frac{w(ps^{t_1})}{w(s^{t_1})}, \tag{20}$$

$$\tilde{\rho}(t;t_1) = w(s^{t_1})w(s^{t-t_1})\rho(t). \tag{21}$$

Figure 4: Resolution Timing of Prospect Risk



The figure shows three different timings of the resolution of prospect risk. Uncertainty gets resolved either immediately at $t = 0$ (left tree), intermediately at t_1 between the present and the time of payment (middle tree), or late at the time of payment at $t > 0$ (right tree). Without loss of generality, we set $\underline{x} = 0$ here.

PROPOSITION 5

Given subproportionality of w, $s < 1$, $t_1 < t$, and folding back:

1. Prospects with prospect risk resolving at the time of payment t are valued more highly than prospects resolving at $t_1 < t$.

2. The wedge between late and immediate resolution, $\frac{w(ps^t)}{w(p)w(s^t)}$, declines with probability p.

3. The wedge between late and immediate resolution increases with time horizon t and survival risk $1-s$.

[Proof in Appendix A]

While it is always the case that late resolution at t is preferred to any earlier resolution time t_1, we cannot ascertain that intermediate resolution at $t_1 > 0$ is generally better than immediate resolution at $t_1 = 0$. Due to the ambiguity effect resulting from time partitions of equal length, discussed above, the discount weight $w(s^{t_1})w(s^{t-t_1})$ decreases with t_1 for $t_1 \in \left[0, \frac{t}{2}\right)$ and increases for $t_1 \in \left(\frac{t}{2}, t\right]$, while risk tolerance increases throughout on $[0, t_1]$. Therefore, total prospect value increases with t_1 as long as both factors increase, which is always the case for $t_1 > \frac{t}{2}$. Depending on the relative magnitudes of the effects before $\frac{t}{2}$, prospect value may decrease after $t_1 = 0$ for some time. Obviously, this depends on the prospect under consideration.

Examining a simple prospect (x, p) with value $u(x)w(ps^{t_1})w(s^{t-t_1})$ shows that the minimum of the utility weight $w(ps^{t_1})w(s^{t-t_1})$ is attained at $t_1^* = \frac{t}{2} - \frac{\ln(p)}{2\ln(s)}$, which lies below $\frac{t}{2}$. If $t_1^* > 0$, then immediate resolution may be preferred to some later times before $\frac{t}{2}$, otherwise prospect value increases monotonically in resolution time. The latter is the case for $p \leq s^t$. For a given prospect, this condition is more likely to be met for low survival risk and/or short time horizons.

In our view, that subproportional risk preferences induce a preference for late resolution of prospect risk constitutes the third important result besides delay- and process-dependence. If decision makers perceive the future as inherently risky and apply folding back, this property follows endogenously from subproportionality and does not constitute an independent preference as in the theoretical literature on resolution timing (Kreps and Porteus, 1978; Chew and Epstein, 1989; Grant, Kajii, and Polak, 2000). Moreover, our model not only predicts a general preference for late resolution of prospect risk, it also specifically addresses skewness preferences because the effect is larger for small probabilities. Additionally, this preference for late resolution of uncertainty of positively skewed prospects increases with time delay.

3.4 Fact # 6: Risk Dependence of Patience

Researchers have been puzzled not only by delay-dependent risk tolerance and preferences with respect to resolution timing but also by other interactions between time and risk, encompassing risk-dependent discounting and diminishing immediacy. As we will show below, these findings can be naturally accommodated within our framework.

Several studies have found that decision makers appear to discount certain future outcomes more heavily than risky ones (Stevenson, 1992; Ahlbrecht and Weber, 1997). Without loss of generality, we analyze two-outcome prospects in this section, $m = 2$. Let V_0 denote the *present value* of the prospect $P = (x_1, p; x_2)$ delayed by t periods. Hence, for $\rho = 1$,

$$V_0 = \left(\left(u(x_1) - u(x_2) \right) \frac{w(ps^t)}{w(s^t)} + u(x_2) \right) w(s^t) \tag{22}$$

Furthermore, let V_t denote the *future value* of P as of t:

$$V_t = \Big(u(x_1) - u(x_2)\Big)w(p) + u(x_2). \tag{23}$$

Discounting by $w(s^t)$ yields

$$V_t w(s^t) = \Big(\Big(u(x_1) - u(x_2)\Big)w(p) + u(x_2)\Big)w(s^t). \tag{24}$$

According to standard discounting theory, the present value V_0 should be equal to the discounted value of V_t, namely $V_t w(s^t)$. However, because $w(p) < \frac{w(ps^t)}{w(s^t)}$, actually $V_t w(s^t) < V_0$. Therefore, it seems as if the certain value V_t is discounted more heavily than the (at t equally attractive) future prospect. The difference in the valuations is not caused by different rates of time preference for risky and certain payoffs, however, but by survival risk changing the nature of the future prospect when evaluated from the point of view of the present rather than from the point of view of the future.

The same kind of risk dependence is at work when the revealed preference for a certain smaller present payoff over an allegedly certain larger later payoff decreases substantially when both payoffs are made (objectively) probabilistic, diminishing immediacy (Keren and Roelofsma, 1995; Weber and Chapman, 2005). Because of the certainty effect, the additional layer of riskiness affects the later payoff much less than the present one because, due to survival risk, it is viewed as a risky prospect already from the outset.

3.5 Fact #7: Order Dependence of Risk Tolerance

An analysis equivalent to risk dependence can be applied to the issue of order dependence of prospect valuation. In principle, there are three different methods of establishing a decision maker's value of a prospect $P = (x_1, p; x_2)$ delayed by t periods: the time-first order, the risk-first order, or the direct method. The time-first order encompasses, at the first stage, the elicitation of the present risky prospect which is considered to be equivalent to the future one and, at the second stage, the elicitation of the certainty equivalent of this present risky prospect. The risk-first order reverses the elicitation stages and assesses the certainty equivalent as of time t first and its present value thereafter. The direct method, finally, elicits the present certainty equivalent of the delayed prospect in one single operation.

When the decision maker is required to state the prospect's value when discounting solely for risk, she ignores the dimension of time and reports V_t, the value of which gets discounted to $V_t w(s^t)$. Conversely, when discounting for time first, she states the present prospect which is equivalent to the delayed one, evaluated as $\Big(\Big(u(x_1) - u(x_2)\Big)\frac{w(ps^t)}{w(s^t)} + u(x_2)\Big)w(s^t)$. Discounting for risk at the second stage results in its value V_0, which is equal to the present value elicited by the direct method.

Therefore, we predict that discounting for risk first results in a lower prospect valuation than discounting for time first. Moreover, discounting for time first is equivalent to prospect evaluation in one single operation. In their study on order dependence, Öncüler and Onay (2009) indeed found this pattern: While valuations resulting from the time-risk order and the direct method are not statistically distinguishable from each other, risk-time evaluations are significantly lower than the ones obtained from the other two methods (see also Ahlbrecht and Weber (1997)).

4 Subproportionality and Decreasing Impatience

In this section we deal with the implications of specific types of probability weighting functions for observed discounting behavior. The most widely used specification of a globally subproportional probability weighting function is Prelec (1998)'s *"standard"* two-parameter specification. While this standard specification has been extensively used in experimental research, Prelec (1998)'s discussion of other forms has been largely ignored by experimental economics. His paper provides a wealth of insights into three different patterns of subproportionality, all of which are derived from preference conditions.[16] As it will become apparent, in our framework these types of subproportional probability weighting functions transfer nicely to well-known specifications of discount functions. Additionally, we examine neo-additive weights, which are particularly useful in the context of ambiguity.

Our discussion centers on the measurement of the degree of subproportionality. As mentioned above, subproportionality reflects convexity of the probability weighting function in log-log coordinates, i.e. convexity of $\omega(\ln p) = \ln\Big(w(p)\Big)$. Therefore, one can define degrees of relative and absolute subproportionality in terms of the Arrow-Pratt indices of convexity of ω, $RC(\omega)$ and $AC(\omega)$:

$$\begin{aligned} RC(\omega(\ln p)) &= -\ln(p)\frac{\omega''}{\omega'}\,, \\ AC(\omega(\ln p)) &= -\frac{\omega''}{\omega'}\,. \end{aligned} \tag{25}$$

While this practice seems straightforward, measuring the degree of departure from constant discounting does not have such an obvious solution. Prelec (2004) discusses several possibilities: First, one could compare discount functions directly by their convexity because convexity promotes higher discounting of the immediate future and lower discounting of the more remote future. Second, one could examine the decline of the discount rate $-\tilde{\rho}'(t)/\tilde{\rho}(t)$ or, finally, one could calculate the degree of convexity of $\ln\tilde{\rho}(t)$, which is equivalent to examining proportional rather than absolute changes in the discount rate. Prelec argues in favor of the third solution because higher convexity of $\ln\tilde{\rho}(t)$ is associated with more choices of dominated options in two-stage

[16] Luce (2001) provides preference conditions for a general class of (not necessarily subproportional) probability weighting functions of the form $w(p) = \exp\Big(-f(p)^{\alpha}\Big)$ which encompasses all three types of Prelec's functions as special cases.

problems. In the following, we adopt this view as well and define the degree of decreasing impatience by the (relative and absolute) Arrow-Pratt indices of convexity of $\ln \tilde{\rho}(t)$ (see also Rohde (2018)).

$$
\begin{aligned}
RC\Big(\ln \tilde{\rho}(t)\Big) &= -t \frac{\Big(\ln \tilde{\rho}(t)\Big)''}{\Big(\ln \tilde{\rho}(t)\Big)'} , \\
AC\Big(\ln \tilde{\rho}(t)\Big) &= -\frac{\Big(\ln \tilde{\rho}(t)\Big)''}{\Big(\ln \tilde{\rho}(t)\Big)'} .
\end{aligned}
\tag{26}
$$

We analyze four cases characterized by different patterns of subproportionality/ convexity of the probability weights. For ease of exposition we set $\rho = 1$.[17]

4.1 Constant Relative/Decreasing Absolute Convexity

The standard Prelec specification of probability weights is given by

$$
w(p) = \exp\Big(-\beta(-\ln p)^{\alpha}\Big) ,
$$

where $\alpha > 0$ determines the degree of departure from linearity. The parameter $\beta > 0$ is a net-index of convexity of w which essentially governs the point of intersection of w with the diagonal. If $\alpha < (=, >)1$, w is subproportional (proportional, supraproportional).

Let us first examine in which sense the parameter α is an index of subproportionality of w. Changing to log-log coordinates, its corresponding ω-function is defined by

$$
\omega(\ln p) = -\beta(-\ln p)^{\alpha} ,
$$

which is a convex function of $\ln p$ as $\omega'' = \alpha(1-\alpha)\beta(-\ln p)^{\alpha-2} > 0$ in the relevant case of $\alpha < 1$. Table 3 summarizes the indices of convexity for all the cases discussed in this section. The first column of Table 3 shows the indices resulting for the standard specification. While relative convexity of ω is constant and equal to $1-\alpha$, its absolute convexity is decreasing. In other words, the standard Prelec probability weighting function with $\alpha < 1$ exhibits constant relative subproportionality $1-\alpha$ and decreasing absolute subproportionality.

Prelec derives this type of subproportional weighting function from the preference axiom of *Compound Invariance*:[18]

The preference relation $\succeq$ exhibits *compound invariance* if for any outcomes $x, y, x', y' \in X$, probabilities $p, q, r, v \in [0,1]$, and compounding integer $N \geq 1$: If $(x, p) \sim (y, q)$ and $(x, r) \sim$

[17] Results also apply to $\rho(t) = \exp(-\eta t)$.

[18] All Prelec (1998)'s results are based on the assumption of a separable representation of the form $V\Big((x,p)\Big) = u(x)w(p)$ for simple prospects (x, p) (a full rank-dependent representation is only required for the one-parameter version of the compound invariant weighting function).

(y, v) then $(x', p^N) \sim (y', q^N)$ implies $(x', r^N) \sim (y', v^N)$.

This condition entails that common ratio violations are preserved under probability discounting, which may be the case when ultimate success requires several independent successes in a row (Prelec (1998), p. 502f).

Let us now turn to the discount function $\tilde{\rho}$ implied by a compound invariant probability weighting function. Substituting s^t for p yields

$$\tilde{\rho}(t) = w(s^t) = \exp\left(-\beta\Big(t(-\ln s)\Big)^{\alpha}\right) = \exp\left(-\gamma t^{1-\theta}\right), \tag{27}$$

where $\gamma = \beta(-\ln s)^{\alpha}$ and $\theta = 1 - \alpha$. This functional form of $\rho(t)$ belongs to the class of Constant Relative Decreasing Impatience (CRDI) discount functions characterized by a constant Arrow-Pratt index of relative log-convexity θ. This class is defined as (Bleichrodt, Rohde, and Wakker, 2009): The discount function $\tilde{\rho}(t)$ is a CRDI function on a time interval $T \subset [0, \infty]$ if there exist constants $\gamma > 0$ and θ, and a normalization constant $k > 0$, such that

$$\tilde{\rho}(t) = \begin{cases} k \exp\left(-\gamma t^{1-\theta}\right) & \text{for } \theta < 1, \\ k t^{-\gamma} & \text{for } \theta = 1 \quad \text{(only if } 0 \notin T), \\ k \exp\left(\gamma t^{1-\theta}\right) & \text{for } \theta > 1 \quad \text{(only if } 0 \notin T). \end{cases} \tag{28}$$

This functional form is quite flexible since it can accommodate decreasing, constant and increasing impatience, depending on the value of the parameter θ. The interesting case is $\theta < 1$ which generates decreasing impatience. By and large, the parameter γ can be interpreted as *level* of impatience.[19] The compound invariant probability weighting function therefore produces a CRDI discount function with normalization $k = 1$ and decreasing impatience as $\theta = 1 - \alpha < 1$. Thus, the degree of relative subproportionality translates directly into the degree of relative decreasing impatience.[20] As Table 3 shows, neither RC nor AC depend on survival risk s. However, s impacts the level of impatience γ. As $\frac{\partial \gamma}{\partial s} = \frac{-\alpha\beta}{s(-\ln s)^{1-\alpha}} < 0$, greater subjective uncertainty, $1 - s$, leads to higher impatience.

[19] Another variant of a CRDI function, the constant-sensitivity discount function $\rho(t) = \exp\left(-(\gamma t)^{\theta}\right)$, which allows for a cleaner separation of time insensitivity θ and impatience γ, was axiomatized by Ebert and Prelec (2007).

[20] Bleichrodt, Rohde, and Wakker (2009) provide a preference condition for CRDI in the case of separable representations. As the compounding of probability p by N translates into $p^N = (s^t)^N = s^{tN}$, not surprisingly, their essentially equivalent axiom involves multiplication of t by a constant factor (N does not have to be an integer, see Prelec (1998)'s proof on p. 517). The authors interpret their preference condition in the following way: If the unit of time is changed, for example from a day to a week, without changing the numbers, then equality of time delays in terms of required outcome compensation should be maintained.

Table 3: Convexity of Probability Weights and Discount Functions

Preference condition	Compound Invariance	Conditional Invariance	Projection Invariance	Uniform Invariance
$RC(\omega)$	$1-\alpha$	$\alpha(-\ln p)$	$\frac{\alpha(-\ln p)}{1-\alpha \ln p}$	$\frac{-\beta \ln p}{\beta+\alpha p}$
$AC(\omega)$	$\frac{1-\alpha}{\ln p}$	$-\alpha$	$\frac{-\alpha}{1-\alpha \ln p}$	$\frac{-\beta}{\beta+\alpha p}$
$RC(\ln \tilde{\rho})$	$1-\alpha$	$\alpha t(-\ln s)$	$\frac{\alpha t(-\ln s)}{1-\alpha t \ln s}$	$\frac{-\beta t \ln s}{\beta+\alpha s^t}$
$AC(\ln \tilde{\rho})$	$\frac{1-\alpha}{t}$	$\alpha(-\ln s)$	$\frac{\alpha(-\ln s)}{1-\alpha t \ln s}$	$\frac{-\beta \ln s}{\beta+\alpha s^t}$

4.2 Decreasing Relative/Constant Absolute Convexity

Another type of a subproportional probability weighting function is the *exponential-power* specification for $p > 0$

$$w(p) = \exp\left(-\frac{\beta}{\alpha}(1-p^\alpha)\right) = \exp\left(\frac{\beta}{\alpha}\Big(\exp\left(-\alpha(-\ln p)\right) - 1\Big)\right),$$

where $\alpha, \beta > 0$. This function is discontinuous at $p = 0$ and is concave and overweighting on $(0, 1)$ for sufficiently small values of β. It is straightforward to show that the Arrow-Pratt index of absolute convexity of its associated ω-function is constant and equal to $-\alpha$ (see Table 3).[21]

The preference condition for the exponential-power case is *Conditional Invariance*:

The preference relation $\succeq$ exhibits *conditional invariance* if for any outcomes $x, y, x', y' \in X$, probabilities $p, q, r, v \in [0, 1]$, and "conditional" probability $\lambda, 0 < \lambda < 1$: If $(x, p) \sim (y, q)$ and $(x, r) \sim (y, v)$ then $(x', \lambda p) \sim (y', \lambda q)$ implies $(x', \lambda r) \sim (y', \lambda v)$.

In this case, common ratio violations are preserved under the proportional reduction of the probabilities, for example pertaining to the case when ultimate success is conditional on some other independent event (Prelec (1998), p. 510).

The corresponding discount function belongs to the class of Constant Absolute Decreasing Impatience (CADI) functions. Following Bleichrodt, Rohde, and Wakker (2009)'s definition:[22]

The discount function $\tilde{\rho}$ is a *CADI function* if there exist constants $\gamma > 0$, θ and a normalization

[21] Prelec (1998) notes that only ω-functions with *decreasing absolute convexity* can exhibit both, subproportionality and continuity at $p = 0$. Note that the full specification in Prelec (1998) (Proposition 4) also includes the (proportional) special case $w(p) = p^\beta$ with $\beta > 0$. The power function cannot accommodate the common ratio effect, and the resulting discount function has an exponential form.

[22] Bleichrodt, Rohde, and Wakker (2009) also provide a preference condition for CADI in the case of separable representations. As multiplying a probability p with a factor $\lambda := s^\tau$ translates into $\lambda p = s^\tau s^t = s^{t+\tau}$, their axiom involves additions of time periods. Consequently, adding a common extra delay to every outcome involved should not change the required compensations.

constant $k > 0$, such that:

$$\tilde{\rho}(t) = \begin{cases} k \exp\Big(\gamma \exp(-\theta t)\Big) & \text{for } \theta > 0\,, \\ k \exp(-\gamma t) & \text{for } \theta = 0\,, \\ k \exp\Big(-\gamma \exp(-\theta t)\Big) & \text{for } \theta < 0\,, \end{cases} \tag{29}$$

with $k = \exp(-\beta/\alpha), \gamma = \beta/\alpha$, and $\theta = \alpha(-\ln s)$.

Such a CADI function exhibits decreasing impatience for $\theta = \alpha(-\ln s) > 0$ which is also equal to the index of absolute convexity of $\ln \tilde{\rho}$. In this case, constant absolute subproportionality governs constant absolute decreasing impatience, which is additionally modulated by subjective uncertainty s, as shown in the second column of Table 3. In fact, both *RC* and *AC* react to survival risk. The greater is subjective uncertainty, $1 - s$, the more decreasingly impatient is behavior.

Mirroring the discontinuity of w at zero, the CADI decreasing impatience specification exhibits a potentially undesirable characteristic: For $t \to \infty$, $\tilde{\rho} \to k = \exp(-\beta/\alpha) > 0$, which may or may not make sense in the specific context of application.[23]

4.3 Increasing Relative/Decreasing Absolute Convexity

The third type of a (weakly) subproportional probability weighting function, discussed by Prelec (1998), is the *hyperbolic logarithm* variety. It is specified as

$$w(p) = (1 - \alpha \ln p)^{-\frac{\beta}{\alpha}}\,, \tag{30}$$

with $\alpha, \beta > 0$. This function is typically inverse S-shaped, but may also become concave and overweighting when β is very small. It displays decreasing absolute/increasing relative subproportionality, as the entries in the third column of Table 3 indicate.

Its corresponding discount function is the *generalized hyperbolic* one (Loewenstein and Prelec, 1992):

$$\tilde{\rho}(t) = (1 + \gamma t)^{-\frac{\theta}{\gamma}}\,, \tag{31}$$

where $\gamma = \alpha(-\ln s) > 0$ and $\theta = \beta(-\ln s) > 0$.

The underlying preference condition is *Projection Invariance*:

For any outcomes x, y and probabilities p, q, r, v: $(x, p) \sim (y, q)$ and $(x, rp) \sim (y, vq)$ implies $(x, r^2 p) \sim (y, v^2 q)$.

This is the simplest of the three cases discussed so far as it only involves two indifferences. Indifference is preserved when the probabilities are squared.

Because of its lack of flexibility, Bleichrodt, Rohde, and Wakker (2009) argue against the generalized hyperbolic discount function - contrary to CRDI and CADI, the hyperbolic specification accommodates increasing impatience only in a particular subcase over a limited time interval and, furthermore, is characterized by an upper bound to the degree of decreasing impatience.

[23]This potentially unattractive feature disappears in the full model with $\tilde{\rho}(t) = w(s^t)\exp(-\eta t)$.

4.4 Decreasing Relative/Increasing Absolute Convexity

An example of decreasing relative and increasing absolute convexity of the ω-function is the neo-additive specification defined in Equation 19. Neo-additive capacities, the equivalent of neo-additive probability weights in decision under ambiguity, were axiomatized by Girotto and Holzer (2016). Their preference condition of *Uniform Invariance* translates to the case of risk as follows:

For any probabilities p_1, p_2, q with $0 < p_i, p_i + q < 1$, $0 \leq q < 1$, $i \in \{1, 2\}$,
$w(p_1 + q) - w(p_1) = w(p_2 + q) - w(p_2)$ holds.

This condition simply means that adding a constant probability increases probability weights uniformly.

The associated discount function is defined as

$$\tilde{\rho}(t) = \begin{cases} 1 & \text{for } t = 0 \\ \beta + \alpha \exp\Big(-(-\ln s)t\Big) & \text{for } t > 0 \end{cases} . \tag{32}$$

For t tending to infinity, $\tilde{\rho}$ tends to β (unless $\rho \neq 1$). For $\beta = 0$, w is not subproportional but captures the certainty effect and, thus, induces quasi-hyperbolic discounting, $\tilde{\rho}(t) = \alpha \exp\Big(-(-\ln s)t\Big)$ (Laibson, 1997). In this case, the parameter $\alpha < 1$ is interpreted as *present bias* or variable costs associated with future payoffs.[24]

A large body of empirical evidence documents the prevalence of common-ratio violations as well as of non-exponential discounting, at least at the level of aggregate behavior (Kahneman and Tversky, 1979; Thaler, 1981; Benzion, Rapoport, and Yagil, 1989; Starmer and Sugden, 1989; Prelec and Loewenstein, 1991). However, there is vast heterogeneity in individuals' behaviors (Hey and Orme, 1994; Chesson and Viscusi, 2000; Bruhin, Fehr-Duda, and Epper, 2010) and the question arises whether common-ratio violations and non-constant discounting are actually exhibited by the same people. To our knowledge, potential links between risk preferences and time preferences at the level of preference conditions or parameter estimates have not been explored so far. However, using the decline of discount rates as a measure of decreasing impatience, Epper, Fehr-Duda, and Bruhin (2011) provide evidence that subjects' departures from linear probability weighting are indeed highly significantly correlated with the strength of the decrease in discount rates. In fact, the only variable associated with decreasing discount rates turns out to be the degree of nonlinearity of probability weights, which explains a large percentage of the variation in the extent of the decline, whereas observable individual characteristics, such as gender, age, experience with investment decisions and cognitive abilities are not significantly correlated with the degree of non-constant discounting. Moreover, Epper, Fehr-Duda, and Bruhin (2011) show

[24] Benhabib, Bisin, and Schotter (2010) find little empirical support for quasi-hyperbolic discounting, but evidence favoring a variant $\rho(t) = -\frac{\beta}{x} + \alpha \exp(-\eta t)$ for $0 < t$, where x denotes the monetary payoff to which discounting is applied. They interpret the term $\frac{\beta}{x}$ as fixed costs that imply both decreasing impatience and the magnitude effect in discounting.

that their data is consistent with a survival probability s of 97% p.a. or higher, quite a realistic value. For example, $s = 99\%$ brings down the implied rate of time preference to 8.5%, a much more plausible value than the high discount rates typically observed in experiments.

5 Implications for Real-World Behavior

A key driver of the seven phenomena described in Table 1 is subproportionality of probability weights, which can take on many different shapes, e.g. convex, concave, or regressive ones. We have chosen a regressive specimen for our graphical illustrations because it is this characteristic that speaks to real world behavior. Regressiveness in the context of RDU produces overweighting of rare extreme events and, thus, a preference for positively skewed distributions and an aversion to negatively skewed ones. This feature has proven useful to make sense of many phenomena in finance and insurance markets. First of all, it solves the classical puzzle of the coexistence of gambling and insuring. In finance, a series of papers found support for Barberis and Huang (2008)'s prediction that assets with higher predicted skewness earn lower returns in the long run (e.g. Ang, Hodrick, Xing, and Zhang (2006); Barberis (2013); Polkovnichenko and Zhao (2013); Boyer and Vorkink (2014); Eraker and Ready (2015)). Concerning the aggregate stock market, De Giorgi and Legg (2012) show that high equity premia and the non-participation puzzle are consistent with the aggregate stock market exhibiting negatively skewed returns. A recent contribution by Dimmock, Kouwenberg, Mitchell, and Peijnenburg (2018) shows that people's decisions whether to participate in the stock market and which stocks to buy seem largely driven by probability weighting. A related finding concerns household portfolios - underdiversified households tend to hold stocks with positively skewed returns (Spalt, 2013).

Extreme risk aversion in insurance markets is another area of application of probability weighting (Sydnor, 2010). For example, Barseghyan, Molinari, O'Donoghue, and Teitelbaum (2013) show that one of the most important drivers of people's deductible choices is the overweighting of small probabilities. Overweighting of small probabilities also explains consumers' purchases of extended warranties for household appliances, and even for telephone wires, at exorbitant prices (Cicchetti and Dubin, 1994; Huysentruyt and Read, 2010). Another example is flight insurance policies for single journeys that used to be extremely popular in the 1950's and 60's when they were sold at vending machines at U.S. airports.[25] Many passengers were willing to pay outrageous premiums, obviously overweighting the rare event of an imminent airplane crash (Meade, 1957).[26]

All these issues relate to the overweighting of rare extreme events, which is predicted by the standard regressive shape of the probability weighting function. However, overweighting of rare extreme events is incompatible with underinsurance which is a prevalent phenomenon as well.

[25] This type of insurance seems to have been popular in other places as well. In 1997 Japanese regulatory agencies granted AIG the right to sell travel insurance via vending machines at Japanese airports, see Fingleton (2008), p. 421.

[26] Taken from Footnote 1 in "Air Trip Insurance", Washington and Lee Law Review 20, Issue 2, Article 16.

For example, inhabitants of disaster-prone areas are often not willing to take insurance even when it is highly subsidized (Kunreuther, 1984; Viscusi, 2010). Globally, between 1960 and 2011 nearly 60% of major natural catastrophes worldwide were uninsured (von Peter, von Dahlen, and Saxena, 2012). Even in high-income countries, only 50% of the damage resulting from catastrophes, such as earthquakes, tsunamis and floods, were covered by insurance contracts. von Peter, von Dahlen, and Saxena (2012) argue that uninsured losses stemming from major natural catastrophes have large and significant negative effects on economic activity, both on impact and over the longer run. Contrary to flight insurance policies for single flights, underinsurance is also prevalent with respect to regular life insurance contracts: Many consumers are reluctant to buy adequate life insurance, thereby exposing their loved ones to considerable poverty risk (Bernheim, Forni, Gokhale, and Kotlikoff, 2003; Cutler, Finkelstein, and McGarry, 2008). This lack of adequate insurance coverage is puzzling because people seem to be extremely risk averse in other domains of economic decisions involving downside risks.

Our model offers a solution to this puzzle. According to our framework, the crucial features of future prospects are the length of delay until outcomes materialize and the process of uncertainty resolution, i.e. whether uncertainty resolves in one shot or sequentially over time. We predict high risk tolerance for long delays combined with one-shot resolution of uncertainty, but comparatively lower risk tolerance for short delays and/or sequentially resolving risks.

Regarding the former type of prospects, consider, for example, natural disasters such as earthquakes and tsunamis. Rarely can their timing be predicted long before their actual occurrence. They literally appear out of the blue. In these cases, uncertainty resolves in one shot at some unknown time in the future. Due to delay dependence of risk tolerance, people display high risk tolerance in these circumstances and, thus, may be unwilling to take out insurance at all.

On the other hand, process dependence of risk tolerance implies that, even for long time horizons, rare adverse events tend to be overweighted when uncertainty resolves sequentially over time. Such a situation applies to the movement of asset prices as price information is readily available, for many assets even in real time. Therefore, notwithstanding the longterm nature of many investments, uncertainty is perceived to resolve sequentially over the course of time rather than in one shot in the distant future which counteracts the risk-tolerance increasing effect of long time delays.

6 Related Literature

In the following we present the experimental evidence in more detail and discuss previous explanations of the observed effects. Extant explanations usually deal with only one or a few specific regularities and do not address the entirety of the phenomena summarized in Table 1. By now, there is an extensive literature on many of these single aspects, for example on hyperbolic discounting, preferences for resolution timing and the value of information. As reviewing this literature is beyond the scope of this paper, we focus on those contributions that are more closely related to our work.

Delay dependence of risk taking behavior, Fact #1 in Table 1, has been documented by a range of papers that do not distinguish between effects of delay on utility and probability weights (Jones and Johnson, 1973; Shelley, 1994; Ahlbrecht and Weber, 1997; Sagristano, Trope, and Liberman, 2002; Noussair and Wu, 2006; Coble and Lusk, 2010). That, in fact, probability weights react to delay, rather than the utility function, was shown experimentally by Abdellaoui, Diecidue, and Öncüler (2011). They conducted a carefully designed experiment eliciting probability weights for both present and delayed prospects, i.e. in our notation $w(p)$ and $\tilde{w}(p)$. Their results provide support for our approach as $\tilde{w}$ is significantly more elevated than w in the aggregate as well as for the majority of the individuals. In their study on ambiguity, Abdellaoui, Baillon, Placido, and Wakker (2011) show estimates of a probability weighting curve derived from choices over prospects delayed by three months. This curve is also much more elevated than typical atemporal estimates are (see for example Bruhin, Fehr-Duda, and Epper (2010)).

While several authors in finance have recognized the importance of accounting for delay-dependent risk tolerance in their models (Khapko, 2015; Eisenbach and Schmalz, 2016), attempts at identifying its potential source are scarce. Baucells and Heukamp (2012) model the delay dependence of risk tolerance in the following way. Aside from their fundamental axiom of a direct probability-time trade-off, mentioned in the introduction, they make two additional assumptions to predict risk premia declining with time delay. The first one is the common ratio effect, which is equivalent to subproportionality in our framework. The second crucial assumption makes the probability-time trade-off depend on outcome magnitude – the probability that renders an early prospect equally attractive as a prospect with a fixed additional delay declines with outcome magnitude. Their approach also predicts hyperbolic discounting (Fact # 2) (for this result, the common ratio effect has to hold as well as decreasing elasticity of the utility function) and the risk dependence of patience (Fact # 6), which is a direct consequence of the probability-time trade-off under subproportionality.

It is well known by now that delay dependence is also manifest in discounting behavior, which constitutes empirical Fact #2. There is abundant evidence that many people exhibit decreasing impatience, i.e. their discount rates are not constant but decline with the length of delay (among many others Benzion, Rapoport, and Yagil (1989); Loewenstein and Thaler (1989); Ainslie (1991); Halevy (2015)). This regularity has triggered a large literature on hyperbolic time preferences and quasi-hyperbolic preferences (e.g. Laibson (1997), for reviews see Frederick, Loewenstein, and O'Donoghue (2002) and Ericson and Laibson (2019)). Most closely related to our approach is Halevy (2008)'s model that derives hyperbolic discounting from the combination of future uncertainty with nonlinear probability weighting. We utilize the same mechanism to derive a large number of additional predictions.

Another regularity in the data concerns the process dependence of risk taking and time discounting behavior, Facts #3 and #4. In the domain of risk, people tend to invest less conservatively, i.e. they take on more risk, when they are informed about the outcomes of their decisions only infrequently (Gneezy and Potters, 1997; Thaler, Tversky, Kahneman, and Schwartz, 1997; Bellemare, Krause, Kröger, and Zhang, 2005; Gneezy, Kapteyn, and Potters, 2003; Haigh and List, 2005).[27] This finding #3 is often interpreted as a manifestation of *myopic loss aversion*, a term coined by Benartzi and Thaler (1995). In this context, myopia is defined as narrow framing of decision situations which focuses on short-term consequences rather than on long-term ones. Loss aversion, one of the key constituents of Prospect Theory, describes people's tendency to be more sensitive to losses than to gains. According to this interpretation, if people evaluate their portfolios frequently, the probability of observing a loss is much greater than if they do so infrequently. While loss aversion may contribute substantially to people's risk aversion, it cannot account for process dependence manifest in decision situations that do not involve any losses.

Process dependence of risk taking was theoretically analyzed in the seminal contributions of Segal who deals with the evaluation of multi-stage prospects in the domain of RDU (Segal, 1987a,b, 1990). Dillenberger (2010) provides a necessary and sufficient condition for preferences for one-shot resolution of uncertainty which holds in Gul (1991)'s model of disappointment aversion, but generally not in RDU. However, we show in Appendix B.1 that this preference condition also applies to the class of prospects studied here.

In the domain of time discounting, a similar phenomenon of process dependence has been observed: The discounting shown over a particular delay is greater when the delay is divided into subintervals than when it is left undivided (Read, 2001; Read and Roelofsma, 2003; Ebert and Prelec, 2007; Epper, Fehr-Duda, and Bruhin, 2009; Dohmen, Falk, Huffman, and Sunde, 2012, 2017). This regularity of subadditive discounting has usually been interpreted as a manifestation of (pure) time preferences.

Fact #5 refers to the effect of the timing of uncertainty resolution on risk taking behavior. Several experimental studies investigated people's intrinsic preferences for resolution timing. Generally, there is a sizable percentage of people with a preference for late resolution (Chew and Ho, 1994; Ahlbrecht and Weber, 1996; Arai, 1997; Lovallo and Kahneman, 2000; Eliaz and Schotter, 2007; von Gaudecker, van Soest, and Wengström, 2011; Ganguly and Tasoff, 2017). Epstein and Zin (1991) also find a preference for late resolution of uncertainty in market data on U.S. consumption and asset returns. In line with our predictions, preference for late resolution seems to be particularly pronounced for positively skewed distributions, i.e. for prospects with small probabilities of the best outcome, and increases with time delay - a prediction that is a distinguishing feature of our model. An intrinsic preference for resolution timing cannot be accommodated by the standard theory of risk taking but is usually modeled by an additional preference parameter (Kreps and Porteus, 1978; Chew and Epstein, 1989; Grant, Kajii, and Polak, 2000). Epstein and Kopylov (2007) and Epstein (2008)' axiomatic papers analyze resolution timing as well. In their work, decision makers may become more pessimistic as payoff time approaches, either due to changes in beliefs or anticipatory feelings (see also Köszegi and Rabin (2009) and Caplin and Leahy (2001)).

[27] In these experiments subjects evaluate *sequences* of identical two-outcome lotteries over several periods where the range of potential outcomes increases with the number of periods. Unlike Gul (1991)'s disappointment aversion (Palacios-Huerta, 1999; Artstein-Avidan and Dillenberger, 2015), our model does not deliver clear predictions for this class of prospects.

Fact #6 pertains to a number of experimental studies that report systematic effects of risk on discounting behavior: Discount rates for certain future payoffs tend to be higher than discount rates for risky future payoffs (Stevenson, 1992; Ahlbrecht and Weber, 1997). Abdellaoui, Kemel, Panin, and Vieider (2017) also find more pronounced discounting for riskless prospects than for risky ones. In their experiment, this gap is substantially narrowed by accounting for nonlinear probability weighting. Risk-dependent discounting is also evident in diminishing immediacy: People's preference for present certain outcomes over delayed ones, immediacy, weakens drastically when the outcomes become risky - they behave as if they discounted the risky reward less heavily than the original certain one (Keren and Roelofsma, 1995; Weber and Chapman, 2005; Baucells and Heukamp, 2010). This evidence motivated Halevy (2008)'s conjecture that future uncertainty might be a driver of this phenomenon. Furthermore, the valuation of future prospects appears to be order dependent: It makes a difference whether a risky future payoff is first devalued for risk and then for delay or in the opposite order (Öncüler and Onay, 2009). When payoffs are discounted for risk first they are assigned a less favorable value than in the reverse case. Moreover, the delay-first value practically coincides with the value reported when both dimensions are accounted for in one single operation. This finding #7 can be also interpreted as a manifestation of risk dependence of discounting.

7 Concluding Remarks

Our framework not only organizes a large number of experimental findings but also speaks to puzzling phenomena observed in real-word behavior. In our view, the most important insights of our approach concern the sensitivity of observed risk tolerance to the length of delay and the way uncertainty resolves. Thus, our model can explain the co-existence of overinsuring and underinsuring as well as the simultaneous occurrence of high equity premia and risk premia declining with maturity.

The ultimate driver of our results is the Allais-type probability dependence of risk attitudes, subproportionality, in conjunction with survival risk, the probability that something may go wrong in the future. In our view, it is hard to deny that the future is inherently uncertain. Presumably, almost everyone has experienced at some time in their lives that something went wrong. Probability dependence of risk preferences, on the other hand, has received overwhelming support from the lab and, increasingly, from the field as well. Thus, our assumptions provide a fertile ground for modeling the intertwined nature of risk taking and discounting behavior.

Our framework provides a host of predictions that can be investigated in future research. Focusing on the first crucial factor, subproportionality, the magnitude of predicted effects depends on its degree. For example, people with comparatively stronger subproportional probability weights should, *ceteris paribus*, exhibit a greater increase in risk tolerance for delayed prospects than less subproportional decision makers do. Similarly, the former group should show a greater preference for uncertainty to resolve in the future rather than in the present. Moreover, these effects are predicted to be more pronounced for positively skewed prospects - a prediction that is specific to our model.

Studying the relationship between risk taking and time discounting constitutes another area for which subproportionality is important. One could examine risk taking and time discounting at the level of preference conditions, as set out in Section 4, to find out whether the same subjects belong to a specific preference type in both decision domains. Alternatively, one could collect rich data on multi-outcome prospects and estimate structural models. Section 4 delineates the expected correlations between risk taking and time discounting parameters. If such correlations are found, they could also indicate the presence of common psychological principles of prospect valuation as suggested by Prelec and Loewenstein (1991). The ultimate test of our model, however, is to exogenously manipulate the subjective probability that something may go wrong, s, the second crucial component of our approach. As effect sizes also depend on the perceived uncertainty of the future, such a manipulation can shed light on the question whether our model has identified an important causal driver of behavior.

Appendix A: Proofs of Propositions

Proof of Proposition 1

1. Since $\tilde{w}(0) = \frac{w(0)}{w(s^t)} = 0$, $\tilde{w}(1) = \frac{w(s^t)}{w(s^t)} = 1$, and $\tilde{w}' = \frac{w'(ps^t)s^t}{w(s^t)} > 0$ hold, $\tilde{w}$ is a proper probability weighting function.

2. Subproportionality of $\tilde{w}$ follows directly from subproportionality of w as for $p > q$ and $0 < \lambda < 1$:
$$\frac{\tilde{w}(\lambda p)}{\tilde{w}(\lambda q)} = \frac{w(\lambda s^t p)}{w(\lambda s^t q)} < \frac{w(s^t p)}{w(s^t q)} = \frac{\tilde{w}(p)}{\tilde{w}(q)} . \tag{33}$$

3.
   - Since w is subproportional,
$$\tilde{w}(p) = \frac{w(ps^t)}{w(s^t)} > \frac{w(p)}{w(1)} = w(p) \tag{34}$$
     holds for $s < 1$ and $t > 0$. Therefore, $\tilde{w}$ is more elevated than w.
   - Obviously, elevation gets progressively higher with increasing t and an equivalent effect is produced by decreasing s. Since $\tilde{w}$ increases monotonically in t and $\tilde{w} \leq 1$ for any t, elevation increases at a decreasing rate.
   - In order to show that a comparatively more subproportional probability weighting function entails a greater increase in observed risk tolerance we examine the relationship between the underlying atemporal probability weights w and observed ones $\tilde{w}$. Let w_1 and w_2 denote two probability weighting functions, with w_2 exhibiting greater subproportionality.

     If $w_1(\lambda)w_1(p) = w_1(\lambda pq)$ holds for a probability $q < 1$, then $w_2(\lambda)w_2(p) < w_2(\lambda pq)$ follows as w_2 is more subproportional than w_1 (Prelec, 1998). Choose $r < 1$ such that $w_2(\lambda)w_2(p) = w_2(\lambda pqr)$. For $\lambda = s^t$, the following relationships hold:
$$\frac{\tilde{w}_1(p)}{w_1(p)} = \frac{w_1(\lambda p)}{w_1(\lambda)w_1(p)} = \frac{w_1(\lambda p)}{w_1(\lambda pq)} . \tag{35}$$
     Applying the same logic to w_2 yields
$$\frac{\tilde{w}_2(p)}{w_2(p)} = \frac{w_2(\lambda p)}{w_2(\lambda)w_2(p)} = \frac{w_2(\lambda p)}{w_2(\lambda pqr)} > \frac{w_2(\lambda p)}{w_2(\lambda pq)} . \tag{36}$$
     Therefore, the relative wedge $\frac{\tilde{w}_2(p)}{w_2(p)}$ caused by subproportionality is larger than the corresponding one for w_1.

4. It is straightforward to show that $\frac{\partial(\tilde{w}(p)/w(p))}{\partial p} = \frac{w(ps^t)}{pw(s^t)w(p)}[\varepsilon_w(ps^t) - \varepsilon_w(p)] < 0$, as the elasticity of a subproportional w, ε_w, is increasing in p.

5. For the elasticity of $\tilde{w}$, $\varepsilon_{\tilde{w}}(p)$, the following relationship holds:

$$\varepsilon_{\tilde{w}}(p) = \frac{\tilde{w}'(p)p}{\tilde{w}(p)} = \frac{w'(ps^t)ps^t}{w(ps^t)} = \varepsilon_w(ps^t) < \varepsilon_w(p)\,, \tag{37}$$

as the elasticity ε_w increases in its argument iff w is subproportional.

6. As $\tilde{w}(p) > w(p)$ holds for any $0 < p < 1$, $\tilde{\pi}_m = 1 - \tilde{w}(1-p_m) < 1 - w(1-p_m) = \pi_m$ results for the decision weight of x_m. As $\tilde{w}$ increases with t, the weight of x_m declines with time delay. ■

Proof of Proposition 2

1. $\tilde{\rho}(0) = w(s^0)\rho^0 = 1$. Since $w' > 0$ holds, $\frac{\partial w(s^t)}{\partial t} < 0$ and, therefore, $\tilde{\rho}' < 0$. Finally, $\lim_{t\to\infty} \tilde{\rho}(t) = 0$ (in terms of discount rates: $\lim_{t\to\infty} \tilde{\eta}(t) = \eta$).

2. Discount rates are generally defined as the rates of decline of the respective discount functions, i.e. $\eta = -\frac{\rho'(t)}{\rho(t)}$ and $\tilde{\eta}(t) = -\frac{\tilde{\rho}'(t)}{\tilde{\rho}(t)}$. Therefore,

$$\begin{aligned}
\tilde{\eta}(t) &= -\frac{\tilde{\rho}'(t)}{\tilde{\rho}(t)} \\
&= -\frac{w'(s^t)s^t \ln(s)\exp(-\eta t) - w(s^t)\exp(-\eta t)\eta}{w(s^t)\exp(-\eta t)} \\
&= -\left(\frac{w'(s^t)s^t}{w(s^t)}\ln(s) - \eta\right) \\
&= -\ln(s)\varepsilon_w(s^t) + \eta \\
&> \eta
\end{aligned} \tag{38}$$

since $\ln(s) < 0, w > 0, w' > 0$. Note that $\frac{w'(s^t)}{w(s^t)}s^t$ corresponds to the elasticity of the probability weighting function w evaluated at s^t, $\varepsilon_w(s^t)$.

3. Since the elasticity of a subproportional function is increasing in its argument, the elasticity of $w(s^t)$ is decreasing in t. Thus,

$$\tilde{\eta}'(t) = -\ln(s)\frac{\partial \varepsilon_w(s^t)}{\partial t} < 0. \tag{39}$$

4. In order to derive the effect of increasing survival risk, i.e. decreasing s, we examine the sensitivity of $\frac{\tilde{\rho}(t+1)}{\tilde{\rho}(t)\tilde{\rho}(1)} = \frac{w(s^{t+1})}{w(s)w(s^t)}$, which measures the departure from constant discounting between periods $t+1$ and t, with respect to changing s:

$$
\begin{aligned}
& \frac{\partial}{\partial s}\left(\frac{w(s^{t+1})}{w(s)w(s^t)}\right) \\
= \; & \frac{1}{\left(w(s)w(s^t)\right)^2}\Big((1+t)s^t w(s)w(s^t)w'(s^{t+1}) - ts^{t-1}w(s)w(s^{t+1})w'(s^t) - w(s^t)w(s^{t+1})w'(s)\Big) \\
= \; & \frac{1}{s\left(w(s)w(s^t)\right)^2}\Big((1+t)s^{t+1} w(s)w(s^t)w'(s^{t+1}) - ts^{t}w(s)w(s^{t+1})w'(s^t) - sw(s^t)w(s^{t+1})w'(s)\Big) \\
= \; & \frac{w(s^{t+1})}{sw(s)w(s^t)}\left(\frac{(1+t)s^{t+1}w'(s^{t+1})}{w(s^{t+1})} - \frac{ts^t w'(s^t)}{w(s^t)} - \frac{sw'(s)}{w(s)}\right) \\
= \; & \frac{w(s^{t+1})}{sw(s)w(s^t)}\Big((1+t)\varepsilon_w(s^{t+1}) - t\varepsilon_w(s^t) - \varepsilon_w(s)\Big) \\
< \; & 0.
\end{aligned}
$$

As $s^{t+1} < s^t < s$, $\varepsilon_w(s^{t+1}) < \varepsilon_w(s^t) < \varepsilon_w(s)$ and, hence, the sum of the elasticities in the final line of the derivation is negative. Therefore, increasing survival risk, i.e. decreasing s, entails a greater departure from constant discounting.

5. In order to examine the effect of the degree of subproportionality on decreasing impatience, suppose that the probability weighting function w_2 is comparatively more subproportional than w_1, as defined in Prelec (1998), and that the following indifference relations hold for two decision makers 1 and 2 at periods 0 and 1:

$$
\begin{aligned}
u_1(y) &= u_1(x)w_1(s)\rho \quad \text{for } 0 < y < x, \\
u_2(y') &= u_2(x')w_2(s)\rho \quad \text{for } 0 < y' < x'\,.
\end{aligned} \tag{40}
$$

Due to subproportionality, the following relation holds for decision maker 1 in period t:

$$
1 = \frac{u_1(x)w_1(s)\rho}{u_1(y)} < \frac{u_1(x)w_1(s^{t+1})\rho^{t+1}}{u_1(y)w_1(s^t)\rho^t}. \tag{41}
$$

Therefore, the probability of prospect survival has to be reduced by compounding s over an additional time period Δt to re-establish indifference:

$$
u_1(y)w_1(s^t)\rho^t = u_1(x)w_1(s^{t+1+\Delta t})\rho^{t+1}. \tag{42}
$$

It follows from the definition of comparative subproportionality that this adjustment of the survival probability by Δt is not sufficient to re-establish indifference with respect to w_2, i.e.

$$
u_2(y')w_2(s^t)\rho^t < u_2(x')w_2(s^{t+1+\Delta t})\rho^{t+1}. \;\blacksquare \tag{43}
$$

Proof of Proposition 3

1. Setting $q = ps^t$ or $q = s^t$, respectively, we prove by induction that $w(q) > \prod_{i=1}^{n} w(q_i)$ for probability q, $0 < q < 1$, and $q = \prod_{i=1}^{n} q_i$.

   - For $n = 2$ subproportionality implies $w(q) = w(q_1 q_2) > w(q_1)w(q_2)$.
   - Assume that $w(\prod_{i=1}^{n} r_i) > \prod_{i=1}^{n} w(r_i)$ for any probabilities $0 < r_i < 1$.
   - For $q = \prod_{j=1}^{n+1} q_j$ subproportionality implies

$$w(q) = w\left(q_{n+1}\prod_{i=1}^{n} q_i\right) > w(q_{n+1})w\left(\prod_{i=1}^{n} q_i\right) > w(q_{n+1})\prod_{i=1}^{n} w(q_i) = \prod_{j=1}^{n+1} w(q_j).$$

2. Without loss of generality, we reorder the sequence of subintervals such that $\tau_1 \leq \tau_2 \leq ... \leq \tau_n$. For some $i, \tau_{i-1} < \tau_i$ holds because otherwise the partition would be equally spaced right away. In this case, there exists $\varepsilon > 0$ such that $\tau_{i-1} + \varepsilon < \tau_i - \varepsilon$ is still satisfied. Due to subproportionality, the following relationship holds for $0 < q < 1$:

$$\frac{w(q^{\tau_{i-1}})}{w(q^{\tau_i - \varepsilon})} > \frac{w(q^{\tau_{i-1}}q^{\varepsilon})}{w(q^{\tau_i - \varepsilon}q^{\varepsilon})} = \frac{w(q^{\tau_{i-1}+\varepsilon})}{w(q^{\tau_i})}\,, \tag{44}$$

   implying $w(q^{\tau_{i-1}})w(q^{\tau_i}) > w(q^{\tau_i - \varepsilon})w(q^{\tau_{i-1}+\varepsilon})$.

3. Consider two equally spaced partitions of $[0, t]$: $(\tau_i = \frac{t}{n} =: \tau)_{i=1,...,n}$ and $(\delta_i = \frac{t}{n-1} =: \delta)_{i=1,...,n-1}$. Our claim is that for $0 < p \leq 1$,

$$\prod_{i=1}^{n} w\left(p^{\frac{\tau}{t}} s^{\tau}\right) < \prod_{i=1}^{n-1} w\left(p^{\frac{\delta}{t}} s^{\delta}\right)\,. \tag{45}$$

   Setting $q = \left(p^{\frac{1}{t}} s\right)^{\frac{t}{n(n-1)}}$, we examine whether

$$\left(w\left(q^{n-1}\right)\right)^{n} < \left(w(q^{n})\right)^{n-1}\,. \tag{46}$$

   Proceeding by induction:

   - $n = 2$: Subproportionality implies $\left(w(q)\right)^2 < w\left(q^2\right)$.

- $n = 3$: Subproportionality implies $w\left(q^3\right) > \frac{\left(w(q^2)\right)^2}{w(q)}$. Thus,

$$
\begin{aligned}
\left(w(q^3)\right)^2 >& \frac{\left(w(q^2)\right)^2}{w(q)} \frac{\left(w(q^2)\right)^2}{w(q)} > \frac{\left(w(q^2)\right)^3 w(q^2)}{\left(w(q)\right)^2} \\
>& \frac{\left(w(q^2)\right)^3 \left(w(q)\right)^2}{\left(w(q)\right)^2} = \left(w(q^2)\right)^3
\end{aligned}
\tag{47}
$$

- $n \to n+1$: Suppose that $\left(w(q^{n-1})\right)^n < \left(w(q^n)\right)^{n-1}$ holds. Subproportionality implies $\frac{w(q^{n-1})}{w(q^n)} > \frac{w(q^n)}{w(q^{n+1})}$. Hence,

$$
\begin{aligned}
\left(w(q^{n+1})\right)^n >& \left(\frac{w(q^n)w(q^n)}{w(q^{n-1})}\right)^n = \frac{\left(w(q^n)\right)^{n+1}\left(w(q^n)\right)^{n-1}}{\left(w(q^{n-1})\right)^n} \\
>& \frac{\left(w(q^n)\right)^{n+1}\left(w(q^{n-1})\right)^n}{\left(w(q^{n-1})\right)^n} = \left(w(q^n)\right)^{n+1}
\end{aligned}
\tag{48}
$$

■

Proof of Proposition 4

1. Consider Equation 18:

$$\tilde{w}_n(p,t) = \prod_{i=1}^{n} \tilde{w}\left(p^{\frac{\tau_i}{t}}, \tau_i\right) .$$

Note that $\tilde{w}\left(p^{\frac{\tau_i}{t}}, \tau_i\right) = \frac{w\left(p^{\frac{\tau_i}{t}} s^{\tau_i}\right)}{w(s^{\tau_i})} < \frac{w\left(p^{\frac{\tau_i}{t}} s^{\tau_i} s^{t-\tau_i}\right)}{w\left(s^{\tau_i} s^{t-\tau_i}\right)} = \frac{w\left(p^{\frac{\tau_i}{t}} s^t\right)}{w(s^t)} = \tilde{w}\left(p^{\frac{\tau_i}{t}}, t\right)$.

According to Proposition 1, $\tilde{w}(p,t)$ is subproportional for a fixed length of delay t and, therefore,

$$\tilde{w}_n(p,t) < \prod_{i=1}^{n} \tilde{w}\left(p^{\frac{\tau_i}{t}}, t\right) < \tilde{w}\left(\prod_{i=1}^{n} p^{\frac{\tau_i}{t}}, t\right) = \tilde{w}(p,t) . \tag{49}$$

2. We proceed by induction.

- Consider the case of $n = 2$ and assume that the time interval of length t is divided into two subintervals of lengths τ and $t - \tau$ with $\tau < t/2 < t - \tau$. We compare $\tilde{w}_n$ corresponding to the evenly spaced partition $(t/2, t/2)$ with the respective $\tilde{w}_n$ for $(\tau, t - \tau)$ by examining when

$$\frac{w\Big((p^{1/t}s)^{t/2}\Big)w\Big((p^{1/t}s)^{t/2}\Big)}{w(s^{t/2})w(s^{t/2})} < \frac{w\Big((p^{1/t}s)^{\tau}\Big)w\Big((p^{1/t}s)^{t-\tau}\Big)}{w(s^{\tau})w(s^{t-\tau})}$$

holds. Rearranging terms yields

$$\frac{w\Big((p^{1/t}s)^{t/2}\Big)w\Big((p^{1/t}s)^{t/2}\Big)}{w\Big((p^{1/t}s)^{\tau}\Big)w\Big((p^{1/t}s)^{t-\tau}\Big)} < \frac{w(s^{t/2})w(s^{t/2})}{w(s^{\tau})w(s^{t-\tau})}.$$

Since $p^{1/t}s < s$ for any $0 < p < 1$, this condition amounts to requiring that $\frac{w(q^{t/2})w(q^{t/2})}{w(q^{\tau})w(q^{t-\tau})}$ increases in q, $0 < q < 1$. It is straightforward to show that its derivative with respect to q equals

$$\frac{\partial}{\partial q}\left(\frac{w(q^{t/2})w(q^{t/2})}{w(q^{\tau})w(q^{t-\tau})}\right) = \frac{t\left(w(q^{t/2})\right)^2}{qw(q^{\tau})w(q^{t-\tau})}\Big(\varepsilon_w(q^{t/2}) - \Big(\lambda\varepsilon_w(q^{\tau}) + (1-\lambda)\varepsilon_w(q^{t-\tau})\Big)\Big),$$

where $\lambda = \tau/t$. As $\tau < t/2 < t - \tau$ and $\varepsilon_w(q^{t-\tau}) < \varepsilon_w(q^{t/2}) < \varepsilon_w(q^{\tau})$, the term in the brackets is positive if the elasticity of w, ε_w, is a strictly concave function.

- For $n \geq 2$ the general formula for the derivative reads as

$$\frac{\Big(w(q^{t/n})\Big)^n}{q\prod_{i=1}^n w(q^{\tau_i})}\Big(t\varepsilon_w(q^{t/n}) - \sum_{i=1}^n \tau_i\varepsilon_w(q^{\tau_i})\Big),$$

where $(\tau_i)_{i=1,\dots,n}$ is a partition of the time interval t with $\sum_{i=1}^n \tau_i = t$.

- $n \to n+1$: Assume that for $t > 0$

$$t\varepsilon_w(q^{t/n}) - \sum_{i=1}^n \tau_i\varepsilon_w(q^{\tau_i}) > 0 \tag{50}$$

holds. Define a partition $(\delta_i)_{i=1,\dots,n+1}$ of t as follows:

$$\begin{aligned} \delta_i &= \frac{n\tau_i}{n+1} \quad \text{for } 1 \leq i \leq n \\ \delta_{n+1} &= t - \sum_{i=1}^n \delta_i = \frac{t}{n+1} \end{aligned}$$

Then the following relationships result:

$$\sum_{i=1}^{n+1} \delta_i \varepsilon_w \left(q^{\delta_i}\right) = \sum_{i=1}^{n} \frac{n\tau_i}{n+1} \varepsilon_w \left(q^{\frac{n\tau_i}{n+1}}\right) + \frac{t}{n+1} \varepsilon_w \left(q^{\frac{t}{n+1}}\right)$$

$$t\varepsilon_w \left(q^{\frac{t}{n+1}}\right) - \frac{t}{n+1} \varepsilon_w \left(q^{\frac{t}{n+1}}\right) = \frac{tn}{n+1} \varepsilon_w \left(q^{\frac{t}{n+1}}\right)$$

Since Equation 50 holds for any $t > 0$ and, therefore, also for $\tilde{t} = \frac{tn}{n+1}$ and $\tilde{\tau}_i = \frac{n\tau_i}{n+1}$,

$$\tilde{t}\varepsilon_w \left(q^{\tilde{t}/n}\right) - \sum_{i=1}^{n} \tilde{\tau}_i \varepsilon_w \left(q^{\tilde{\tau}_i}\right) > 0, \tag{51}$$

which implies

$$\frac{tn}{n+1} \varepsilon_w \left(q^{\frac{t}{n+1}}\right) - \sum_{i=1}^{n} \frac{n\tau_i}{n+1} \varepsilon_w \left(q^{\frac{n\tau_i}{n+1}}\right) > 0. \tag{52}$$

3. We examine whether $\left(\frac{w\left((p^{1/t}s)^{t/n}\right)}{w(s^{t/n})}\right)^n < \left(\frac{w\left((p^{1/t}s)^{t/(n-1)}\right)}{w(s^{t/(n-1)})}\right)^{n-1}$, which is equal to the condition that

$$\frac{\left(w\left((p^{1/t}s)^{t/n}\right)\right)^n}{\left(w\left((p^{1/t}s)^{t/(n-1)}\right)\right)^{n-1}} < \frac{\left(w(s^{t/n})\right)^n}{\left(w\left(s^{t/(n-1)}\right)\right)^{n-1}}.$$

Therefore, we examine whether the derivative of $\frac{\left(w(q^{t/n})\right)^n}{\left(w(q^{t/(n-1)})\right)^{n-1}}$ with respect to q is positive. It is straightforward to show that

$$\frac{\partial \frac{\left(w(q^{t/n})\right)^n}{\left(w(q^{t/(n-1)})\right)^{n-1}}}{\partial q} = \frac{t\left(w(q^{t/n})\right)^n}{q\left(w\left(q^{t/(n-1)}\right)\right)^{n-1}} \left(\varepsilon_w\left(q^{t/n}\right) - \varepsilon_w\left(q^{t/(n-1)}\right)\right) > 0 \tag{53}$$

as the elasticity of w is increasing. ■

Proof of Proposition 5

Without loss of generality, we set the number of outcomes $m = 2$.

1. The value of the prospect to be resolved immediately amounts to

$$\begin{aligned} &\Big(\big(u(x_1) - u(x_2)\big)w(p) + u(x_2)\Big)w(s^t) \\ &< \left(\big(u(x_1) - u(x_2)\big)\frac{w(ps^t)}{w(s^t)} + u(x_2)\right) w(s^t), \end{aligned} \tag{54}$$

as $w(ps^t) > w(p)w(s^t)$ is implied by subproportionality of w. Thus, prospects resolving at the date of payment t are valued more highly than prospects with immediate resolution.

What happens if prospect risk is not resolved immediately but rather at some later time t_1, $0 < t_1 < t$? After t_1, only survival risk remains to be resolved. In this case, the prospect's present value amounts to

$$\left(\Big(u(x_1) - u(x_2)\Big)\frac{w(ps^{t_1})}{w(s^{t_1})} + u(x_2)\right) w(s^{t_1})w(s^{t-t_1}). \tag{55}$$

Subproportionality implies $w(p) < \frac{w(ps^{t_1})}{w(s^{t_1})} < \frac{w(ps^t)}{w(s^t)}$ and, therefore, observed risk tolerance is highest for resolution at payment time t. Moreover, the late-resolution discount weight $w(s^t) = w(s^{t_1}s^{t-t_1})$ is also greater than $w(s^{t_1})w(s^{t-t_1})$ for any earlier t_1, implying that late resolution is always preferred.

2. Examining the derivative of $\frac{w(ps^t)}{w(p)}$ with respect to p yields

$$\begin{aligned}\frac{\partial\left(\frac{w(ps^t)}{w(p)}\right)}{\partial p} &= \frac{w(ps^t)}{pw(p)}\left(\frac{w'(ps^t)ps^t}{w(ps^t)} - \frac{w'(p)p}{w(p)}\right)\\ &= \frac{w(ps^t)}{pw(p)}\Big(\varepsilon_w(ps^t) - \varepsilon_w(p)\Big)\\ &< 0,\end{aligned} \tag{56}$$

as $p > ps^t$ and the elasticity is increasing. Therefore, the wedge between late evaluation and immediate evaluation decreases with p.

3. The derivative of $\frac{w(ps^t)}{w(s^t)}$ with respect to t yields

$$\begin{aligned}\frac{\partial\left(\frac{w(ps^t)}{w(s^t)}\right)}{\partial t} &= \frac{\ln(s)w(ps^t)}{w(s^t)}\left(\frac{w'(ps^t)ps^t}{w(ps^t)} - \frac{w'(s^t)s^t}{w(s^t)}\right)\\ &= \frac{\ln(s)w(ps^t)}{w(s^t)}\Big(\varepsilon_w(ps^t) - \varepsilon_w(s^t)\Big)\\ &> 0,\end{aligned} \tag{57}$$

as $\ln(s) < 0$, $s^t > ps^t$ and the elasticity is increasing. Therefore, the wedge between late and immediate evaluation increases with time horizon t and, equivalently, with survival risk $1 - s$. ■

Appendix B: Additional Materials

B.1 A Note on Sequential Evaluation

In his Proposition 1, Dillenberger (2010) shows that, under recursivity, negative certainty independence (NCI) and a weak preference for one-shot resolution of uncertainty (PORU) are equivalent. The NCI axiom requires the following to hold: If a prospect $P = (x_1, r; x_2)$ is weakly preferred to a degenerate prospect $D = (y, 1)$, then mixing both with any other prospect does not result in the mixture of the degenerate prospect D being preferred to the mixture of P. This axiom is weaker than the standard independence axiom and does not put any restrictions on the reverse preference relation when a degenerate prospect is originally preferred to a nondegenerate one. The latter case characterizes the typical Allais certainty effect. NCI allows for Allais-type preference reversals but does not imply them. David Dillenberger's Proposition 3 demonstrates that NCI is generally incompatible with rank-dependent utility unless the probability weighting function is linear, i.e. unless RDU collapses to EUT. An intuitive explanation for Dillenberger's Proposition 3 is that under RDU prospect valuation is sensitive to the rank order of the outcomes and, therefore, mixtures with other prospects may affect the original rank order of outcomes in P (and D). How does Dillenberger's result relate to our claim that subproportional probability weights conjointly with recursivity imply a strong preference for one-shot resolution of uncertainty?

The crucial insight is that for the class of prospects studied in this paper changes in rank order do not occur and, hence, NCI is satisfied. To see this, assume that the prospect $(x_1, p; x_2)$, $x_1 > x_2 \geq 0$, gets resolved in two stages $\Big((x_1, r; x_2), q; (x_2, 1)\Big)$ such that $p = qr$. In the atemporal case, when there is no additional survival risk, the two-stage prospect continues to be a strictly two-outcome one and the only relevant mixtures are those involving x_2. Suppose that $P = (x_1, r; x_2) \succsim (y, 1) = D$, with $x_1 > y > x_2$ and consider the following mixtures with $(x_2, 1 - \lambda)$ for some $\lambda \in (0, 1)$: $P' = (x_1, \lambda r; x_2)$ and $D' = (y, \lambda; x_2)$. The following relationships hold:

$$\begin{aligned}
P \succsim D \quad &\Rightarrow V(P) = \Big(u(x_1) - u(x_2)\Big)w(r) + u(x_2) \geq u(y) = V(D) \\
V(D') \quad &= u(y)w(\lambda) + u(x_2)\Big(1 - w(\lambda)\Big) \\
&\leq \bigg(\Big(u(x_1) - u(x_2)\Big)w(r) + u(x_2)\bigg)w(\lambda) + u(x_2)\Big(1 - w(\lambda)\Big) \\
&= \Big(u(x_2) - u(x_1)\Big)w(r)w(\lambda) + u(x_2) \\
&< \Big(u(x_2) - u(x_1)\Big)w(\lambda r) + u(x_2) \\
&= V(P')
\end{aligned} \tag{58}$$

because $w(r)w(\lambda) < w(\lambda r)$ for any $\lambda \in (0, 1)$ (and hence also for $\lambda = q$) due to subproportionality of w. Consequently, for mixtures with the smaller outcome x_2, NCI, and therefore also PORU, is *strongly* satisfied. If the mixing prospect may be any arbitrary prospect, in other words if surprises are possible in the course of uncertainty resolution, this result does not hold generally.

The only surprise that is still admissible is the occurrence of an outcome worse than x_2, say z. Define $P'' = \Big(x_1, \lambda r; x_2, \lambda(1-r); z\Big)$ and $D'' = (y, \lambda; z)$.

$$\begin{aligned} V(D'') &= u(y)w(\lambda) + u(z)\Big(1 - w(\lambda)\Big) \\ &\le \bigg(\Big(u(x_1) - u(x_2)\Big)w(r) + u(x_2)\bigg)w(\lambda) + u(z)\Big(1 - w(\lambda)\Big) \\ &= \Big(u(x_1) - u(x_2)\Big)w(r)w(\lambda) + \Big(u(x_2) - u(z)\Big)w(\lambda) + u(z) \\ &< \Big(u(x_1) - u(x_2)\Big)w(\lambda r) + \Big(u(x_2) - u(z)\Big)w(\lambda) + u(z) \\ &= V(P'') \end{aligned} \tag{59}$$

For $u(z) = 0$, this case is exactly the situation studied in this paper when survival risk comes into play.

B.2 Characteristics of Functional Specifications of Probability Weights

In this section we review a number of probability weighting functions that are either globally or locally subproportional. We limit our attention to functional forms with at most two parameters. Recall that subproportionality is equivalent to increasing elasticity. Consequently, if the elasticity is U-shaped, the function is supraproportional over the range of small probabilities and subproportional over large probabilities. These functions still capture the certainty effect but not necessarily general common-ratio violations. Many specifications used in the literature exhibit such a characteristic. Some experimenters found reverse common-ratio violations which require supraproportionality over the relevant probability range (see e.g. Blavatskyy (2010)). Ultimately, it is an empirical issue whether locally or globally subproportional functions fit better.

Polynomials are linear in the parameters and, thus, generally less flexible than specifications that are nonlinear in the parameters. Note that second-order polynomials demarcate the intersection of the class of quadratic utility and RDU (see also the discussion in Masatlioglu and Raymond (2016)).

Gul (1991)'s theory of disappointment aversion, for example, implies a strictly convex subproportional function in the context of RDU for two-outcome prospects. Another interesting specimen is the probability weighting function discussed in Delquié and Cillo (2006). In the context of RDU, their model of disappointment aversion generates a subproportional second-order polynomial that is equivalent to the one implied by Köszegi and Rabin (2007)'s choice-acclimating personal equilibrium, which provides an endogenous reference point (Masatlioglu and Raymond, 2016). The same polynomial also emerges in Safra and Segal (1998)'s approach to constant risk aversion. This concept captures the idea that a decision maker commits to a choice long before uncertainty is resolved, and is, therefore, particularly plausible in the context of our model. Under specific assumptions, Bordalo, Gennaioli, and Shleifer (2012) derive (discontinuous) context-dependent probability distortions from their salience theory. While their concave segment is supraproportional, the convex segment is subproportional, both of the Gul (1991) variety with $0 < \beta < 1$ and $\beta > 1$, respectively. The psychological mechanisms underlying probability weighting, therefore, often imply at least some extent of subproportionality.

An intermediate case is the constant-sensitivity specification suggested by Abdellaoui, l'Haridon, and Zank (2010) which is subproportional for large probabilities but exhibits constant elasticity for small probabilities. Thus, risk tolerance increases with delay until it hits an upper bound, staying constant afterwards. Ultimately, it is an open question whether this feature is consistent with actual behavior, which provides a fruitful avenue for future research. In particular, the associated discount function is characterized by decreasing impatience for more imminent time horizons, but constant impatience for more remote horizons. Thus, it constitutes an alternative to the quasi-hyperbolic β-δ model.

Table 4: Probability Weighting Functions

Probability weighting function $\mathbf{w(p)}$	**Parameter range**	**Elasticity***	**Shape****	**Reference**
p^α	$\alpha > 1$	constant	convex	Luce, Mellers, and Chang (1993)
$\frac{p}{2-p}$	-	increasing	convex	Yaari (1987)
$\exp\left(-\beta(-\ln(p))^\alpha\right)$	$0 < \alpha < 1, \beta > 0$	increasing, concave/convex	regressive	Prelec (1998)
	$\alpha = 1, \beta > 1$	constant	convex	Prelec (1998)[1]
$\exp\left(-\frac{\beta}{\alpha}(1-p^\alpha)\right)$	$\alpha, \beta > 0$	increasing	concave, regressive	Prelec (1998)[7]
$(1-\alpha\ln p)^{-\frac{\beta}{\alpha}}$	$\alpha, \beta > 0$	increasing	regressive	Prelec (1998)
$\frac{p^\alpha}{(p^\alpha+(1-p)^\alpha)^{1/\alpha}}$	$0.279 < \alpha < 1$	U-shaped	regressive	Tversky and Kahneman (1992)
$\frac{\beta p^\alpha}{\beta p^\alpha+(1-p)^\alpha}$	$0 < \alpha < 1, \beta > 0$	U-shaped	regressive	Goldstein and Einhorn (1987)
	$0 < \alpha < 1, \beta = 1$	U-shaped	regressive	Karmarkar (1979)
	$\alpha = 1, \beta < 1$	increasing, convex	convex	Rachlin, Raineri, and Cross (1991)
		see text	see text	Bordalo, Gennaioli, and Shleifer (2012)[2]
$\frac{p+\alpha p(1-p)}{1+(\alpha+\beta)p(1-p)}$	$\alpha > 0, \beta > 0$	U-shaped	regressive	Walther (2003)
$\begin{cases} \beta^{1-\alpha}p^\alpha & \text{if (i) } 0 \le p \le \beta \\ 1-(1-\beta)^{1-\alpha}(1-p)^\alpha & \text{if (ii) } \beta < p \le 1 \end{cases}$	$0 < \alpha, \beta < 1$	(i) constant, (ii) increasing	regressive	Abdellaoui, l'Haridon, and Zank (2010)[3]
$\frac{p}{1+(1-p)\beta}$	$\beta > 1$	increasing, convex	convex	Gul (1991)[6]
$p - \alpha p + \alpha p^2$	$0 < \alpha < 1$	increasing, concave	convex	Masatlioglu and Raymond (2016); Delquié and Cillo (2006); Safra and Segal (1998)[4]
$p + \frac{3-3\beta}{\alpha^2-\alpha+1}(\alpha p - (\alpha+1)p^2 + p^3)$	$0 < \alpha, \beta < 1$	U-shaped	regressive	Rieger and Wang (2006)
$p - \alpha p(1-p) + \beta p(1-p)(1-2p)$	α depends on β	variety	variety	Blavatskyy (2014)[5]
$\begin{cases} 0 & \text{for } p = 0 \\ \beta + \alpha p & \text{for } 0 < p < 1 \\ 1 & \text{for } p = 1 \end{cases}$	$0 < \beta < 1$, $0 < \alpha \le 1-\beta$	increasing, concave	regressive	Bell (1985); Cohen (1992); Chateauneuf, Eichberger, and Grant (2007)

∗ Increasing elasticity is equivalent to *subproportionality*. ∗∗ An inverse-S shape means that both tails are overweighted, i.e. that the weighting function is *regressive*.

(1) Equivalent to power specification $w(p) = p^\beta$.

(2) The weighting function consists of a concave and a convex segment with a jump discontinuity in between (see text).

(3) For $\alpha > 1, \beta = 1$ constant elasticity, convex; for $\alpha < 1, \beta = 0$ increasing elasticity, convex.

(4) Special case of Blavatskyy (2014) with $\beta = 0$.

(5) Specific parameter constellations with $\beta > 0$ generate regressive with U-shaped elasticity.

(6) Identical to Rachlin, Raineri, and Cross (1991).

(7) The full specification of the conditional invariant form also contains the power function (see row 1) as a special case (Prelec (1998), Proposition 4).

References

ABDELLAOUI, M., A. BAILLON, L. PLACIDO, AND P. WAKKER (2011): "The Rich Domain of Uncertainty: Source Functions and Their Experimental Implementation," *American Economic Review*, 101, 695–723.

ABDELLAOUI, M., E. DIECIDUE, AND A. ÖNCÜLER (2011): "Risk Preferences at Different Time Periods: An Experimental Investigation," *Management Science*, 57(5), 975–987.

ABDELLAOUI, M., E. KEMEL, A. PANIN, AND F. M. VIEIDER (2017): "Take Your Chance or Take Your Time. On the Impact of Risk on Time Discounting," *Working Paper*.

ABDELLAOUI, M., O. L'HARIDON, AND H. ZANK (2010): "Separating Curvature and Elevation: A Parametric Probability Weighting Function," *Journal of Risk and Uncertainty*, 41, 39–65.

AHLBRECHT, M., AND M. WEBER (1996): "The Resolution of Uncertainty: An Experimental Study," *Journal of Institutional and Theoretical Economics*, 152, 593–607.

——— (1997): "An Empirical Study on Intertemporal Decision Making under Risk," *Management Science*, 43(6), 813–826.

AINSLIE, G. (1991): "Derivation of 'Rational' Economic Behavior from Hyperbolic Discount Curves," *American Economic Review, Papers and Proceedings*, 81(2), 334–340.

ALLAIS, M. (1953): "Le Comportement de L'Homme Rationnel Devant le Risque: Critique des Postulats et Axiomes de L'Ecole Americaine," *Econometrica*, 21, 503–546.

ANDREONI, J., AND C. SPRENGER (2012): "Risk Preferences Are Not Time Preferences," *American Economic Review*, 102(7), 3357–3376.

ANG, A., R. J. HODRICK, Y. XING, AND X. ZHANG (2006): "The Cross-Section of Volatility and Expected Returns," *The Journal of Finance*, 61(1), 259–299.

ARAI, D. (1997): "Temporal Resolution of Uncertainty in Risky Choices," *Acta Psychologica*, 96, 15–26.

ARTSTEIN-AVIDAN, S., AND D. DILLENBERGER (2015): "Dynamic Disappointment Aversion," *Working Paper*.

BAILLON, A., O. L'HARIDON, AND L. PLACIDO (2011): "Ambiguity Models and the Machina Paradoxes," *American Economic Review*, 101(4), 1547–1560.

BARBERIS, N. (2013): "Thirty Years of Prospect Theory in Economics: A Review and Assessment," *Journal of Economic Perspectives*, 27(1), 173–196.

BARBERIS, N., AND M. HUANG (2008): "Stocks as Lotteries: The Implications of Probability Weighting for Security Prices," *American Economic Review*, 98(5), 2066–2100.

Barseghyan, L., F. Molinari, T. O'Donoghue, and J. Teitelbaum (2013): "The Nature of Risk Preferences: Evidence from Insurance Choices," *American Economic Review*, 103, 2499–2529.

Baucells, M., and F. H. Heukamp (2010): "Common Ratio Using Delay," *Theory and Decision*, 68, 149–158.

——— (2012): "Probability and Time Tradeoff," *Management Science*, 58(4), 831–842.

Bell, D. (1985): "Disappointment in Decision Making under Uncertainty," *Operations Research*, 33(1), 1–27.

Bellemare, C., M. Krause, S. Kröger, and C. Zhang (2005): "Myopic Loss Aversion: Information Feedback vs. Investment Flexibility," *Economics Letters*, 87, 319–324.

Benartzi, S., and R. Thaler (1995): "Myopic Loss Aversion and the Equity Premium Puzzle," *Quarterly Journal of Economics*, 110(1), 73–92.

Benhabib, J., A. Bisin, and A. Schotter (2010): "Present-Bias, Quasi-Hyperbolic Discounting, and Fixed Costs," *Games and Economic Behavior*, 69, 205–223.

Benzion, U., A. Rapoport, and J. Yagil (1989): "Discount Rates Inferred from Decisions: An Experimental Study," *Management Science*, 35(3), 270–284.

Bernheim, B. D., L. Forni, J. Gokhale, and L. J. Kotlikoff (2003): "The Mismatch Between Life Insurance Holdings and Financial Vulnerabilities: Evidence from the Health and Retirement Study," *American Economic Review*, 93(1), 354–365.

Blavatskyy, P. (2010): "Reverse Common Ratio Effect," *Journal of Risk and Uncertainty*, 40, 219–241.

Blavatskyy, P. (2014): "A Probability Weighting Function for Cumulative Prospect Theory and Mean-Gini Approach to Optimal Portfolio Investment," *mimeo*.

Bleichrodt, H., K. I. M. Rohde, and P. Wakker (2009): "Non-Hyperbolic Time Inconsistency," *Games and Economic Behavior*, 66, 27–38.

Bommier, A. (2007): "Risk Aversion, Intertemporal Elasticity of Substitution and Correlation Aversion," *Economics Bulletin*, 4(29), 1–8.

Bordalo, P., N. Gennaioli, and A. Shleifer (2012): "Salience Theory of Choice under Risk," *Quarterly Journal of Economics*, 127(3), 1243–1285.

Boyer, B. H., and K. Vorkink (2014): "Stock Options as Lotteries," *The Journal of Finance*, 69(4), 1485–1527.

Bruhin, A., H. Fehr-Duda, and T. Epper (2010): "Risk and Rationality: Uncovering Heterogeneity in Probability Distortion," *Econometrica*, 78(4), 1375–1412.

Caplin, A., and J. Leahy (2001): "Psychological Expected Utility Theory and Anticipatory Feelings," *Quarterly Journal of Economics*, 116(1), 55–79.

Chateauneuf, A., J. Eichberger, and S. Grant (2007): "Choice Under Uncertainty With the Best and Worst in Mind: Neo-additive Capacities," *Journal of Economic Theory*, 137(1), 538–567.

Chateauneuf, A., and P. Wakker (1999): "An Axiomatization of Cumulative Prospect Theory for Decision Under Risk," *Journal of Risk and Uncertainty*.

Chesson, H., and W. Viscusi (2000): "The Heterogeneity of Time-Risk Tradeoffs," *Journal of Behavioral Decision Making*, 13, 251–258.

Chew, S., and L. Epstein (1989): "The Structure of Preferences and Attitudes towards the Timing of the Resolution of Uncertainty," *International Economic Review*, 30(1), 103–117.

Chew, S., and J. Ho (1994): "Hope: An Empirical Study of Attitude Toward the Timing of Uncertainty Resolution," *Journal of Risk and Uncertainty*, 8, 267–288.

Cicchetti, C., and J. Dubin (1994): "A Microeconometric Analysis of Risk Aversion and the Decision to Self-insure," *Journal of Political Economy*, 102(1), 169–186.

Coble, K., and J. Lusk (2010): "At the Nexus of Risk and Time Preferences: An Experimental Investigation," *Journal of Risk and Uncertainty*, 41(1), 67–79.

Cohen, M. (1992): "Security Level, Potential Level, Expected Utility: a Three-criteria Decision Model Under Risk," *Theory and Decision*, 33, 101–134.

Cutler, D. M., A. Finkelstein, and K. McGarry (2008): "Preference Heterogeneity and Insurance Markets: Explaining a Puzzle of Insurance," *American Economic Review*, 98(2), 157–162.

De Giorgi, E., and s. Legg (2012): "Dynamic Portfolio Choice and Asset Pricing with Narrow Framing and Probability Weighting," *Journal of Economic Dynamics and Control*, 36(7), 951–972.

Delquié, P., and A. Cillo (2006): "Disappointment Without Prior Expectation: a Unifying Perspective on Decision Under Risk," *Journal of Risk and Uncertainty*, 33, 197–215.

Denuit, M., L. Eeckhoudt, and B. Rey (2010): "Some Consequences of Correlation Aversion in Decision Science," *Annals of Operations Research*, 176, 259–269.

Diecidue, E., and P. P. Wakker (2001): "On the Intuition of Rank-Dependent Utility," *Journal of Risk and Uncertainty*, 23(3), 281–298.

Dillenberger, D. (2010): "Preferences for One-Shot Resolution of Uncertainty," *Econometrica*, 78(6), 1973–2004.

Dillenberger, D., and U. Segal (2014): "Recursive Ambiguity and Machina's Examples," *International Economic Review*, 56, 55–61.

Dimmock, S. G., R. Kouwenberg, O. S. Mitchell, and K. Peijnenburg (2018): "Household Portfolio Underdiversification and Probability Weighting: Evidence from the Field," *Working Paper*.

Dohmen, T., A. Falk, D. Huffman, and U. Sunde (2012): "Interpreting Time Horizon Effects in Inter-Temporal Choice," *IZA Discussion Paper No. 6385*.

Dohmen, T., A. Falk, D. Huffman, and U. Sunde (2017): "The Robustness and Pervasiveness of Sub-Additivity in Intertemporal Choice," *Working Paper*.

Ebert, J., and D. Prelec (2007): "The Fragility of Time: Time-Insensitivity and Valuation of the Near and Far Future," *Management Science*, 53(9), 1423–1438.

Eisenbach, T. M., and M. C. Schmalz (2016): "Anxiety in the Face of Risk," *Journal of Financial Economics*, 121, 414–426.

Eliaz, K., and A. Schotter (2007): "Experimental Testing of Intrinsic Preferences for Non-Instrumental Information," *American Economic Review, Papers and Proceedings*, 97(2), 166–169.

Epper, T., and H. Fehr-Duda (2015a): "Comment on "Risk Preferences Are Not Time Preferences": Balancing on a Budget Line," *American Economic Review*, 105(7), 2261–2271.

——— (2015b): "The Missing Link: Unifying Risk Taking and Time Discounting," *Working Paper*.

Epper, T., H. Fehr-Duda, and A. Bruhin (2009): "Uncertainty Breeds Decreasing Impatience: The Role of Risk Preferences in Time Discounting," *Working Paper, Institute for Empirical Research in Economics, University of Zurich*, 412.

——— (2011): "Viewing the Future through a Warped Lens: Why Uncertainty Generates Hyperbolic Discounting," *Journal of Risk and Uncertainty*, 43(3), 169–203.

Epstein, L., and S. Tanny (1980): "Increasing Generalized Correlation: a Definition and Some Economic Consequences," *Canadian Journal of Economics*, 13(1), 16–34.

Epstein, L., and S. Zin (1991): "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: An Empirical Analysis," *Journal of Political Economy*, 99(2), 263–286.

Epstein, L. G. (2008): "Living with Risk," *Review of Economic Studies*, 75, 1121–1141.

Epstein, L. G., and I. Kopylov (2007): "Cold Feet," *Theoretical Economics*, 2, 231–259.

Eraker, B., and M. Ready (2015): "Do Investors Overpay for Stocks with Lottery-Like Payoffs? An Investigation of the Returns of OTC Stocks," *Journal of Financial Economics*, 115, 486–504.

Ericson, K. M., and D. Laibson (2019): "Intertemporal Choice," *Bernheim, D., Laibson, D., DellaVigna, S.: Handbook of Behavioral Economics - Foundations and Applications 2, Vol. 2, Elsevier*.

FEHR-DUDA, H., AND T. EPPER (2012): "Probability and Risk: Foundations and Economic Implications of Probability-Dependent Risk Preferences," *Annual Review of Economics*, 4, 567–593.

FINGLETON, E. (2008): *In the Jaws of the Dragon: America's Fate in the Coming Era of Chinese Hegemony*. Thomas Dunne Books, St. Martin's Griffith, NY.

FREDERICK, S., G. F. LOEWENSTEIN, AND T. O'DONOGHUE (2002): "Time Discounting and Time Preference: A Critical Review," *Journal of Economic Literature*, 40(2), 351–401.

GANGULY, A., AND J. TASOFF (2017): "Fantasy and Dread: The Demand for Information and the Consumption Utility of the Future," *Management Science*, 63(12), 4037–4060.

GIROTTO, B., AND S. HOLZER (2016): "A Characterization of Neo-Additive Measures," *Fuzzy Sets and Systems*, 284, 56–62.

GNEEZY, U., A. KAPTEYN, AND J. POTTERS (2003): "Evaluation Periods and Asset Prices in a Market Experiment," *Journal of Finance*, LVIII(2), 821–837.

GNEEZY, U., AND J. POTTERS (1997): "An Experiment on Risk Taking and Evaluation Periods," *Quarterly Journal of Economics*, 112(2), 631–645.

GOLDSTEIN, W., AND H. EINHORN (1987): "Expression Theory and the Preference Reversal Phenomena," *Psychological Review*, 94, 236–254.

GONZALEZ, R., AND G. WU (1999): "On the Shape of the Probability Weighting Function," *Cognitive Psychology*, 38, 129–166.

GRANT, S., A. KAJII, AND B. POLAK (2000): "Temporal Resolution of Uncertainty and Recursive Non-expected Utility Models," *Econometrica*, 68(2), 425–434.

GUL, F. (1991): "A Theory of Disappointment Aversion," *Econometrica*, 59(3), 667–686.

HAIGH, M., AND J. LIST (2005): "Do Professional Traders Exhibit Myopic Loss Aversion? An Experimental Analysis," *Journal of Finance*, 60(1), 523–534.

HALEVY, Y. (2008): "Strotz Meets Allais: Diminishing Impatience and Certainty Effect," *American Economic Review*, 98(3), 1145–1162.

——— (2015): "Time Consistency: Stationarity and Time Invariance," *Econometrica*, 83(1), 335–352.

HEY, J. D., AND C. ORME (1994): "Investigating Generalizations of Expected Utility Theory Using Experimental Data," *Econometrica*, 62(6), 1291–1326.

HUYSENTRUYT, M., AND D. READ (2010): "How Do People Value Extended Warranties? Evidence From Two Field Surveys," *Journal of Risk and Uncertainty*, 40, 197–218.

Jones, E. E., and C. A. Johnson (1973): "Delay of Consequences and the Riskiness of Decisions," *Journal of Personality*, 41(4), 613–637.

Kahneman, D., and A. Tversky (1979): "Prospect Theory: An Analysis of Decision under Risk," *Econometrica*, 47(2), 263–292.

Karmarkar, U. S. (1979): "Subjectively Weighted Utility and the Allais Paradox," *Organizational Behavior and Human Performance*, 24, 67–72.

Keren, G., and P. Roelofsma (1995): "Immediacy and Certainty in Intertemporal Choice," *Organizational Behavior and Human Decision Processes*, 63(3), 287–297.

Khapko, M. (2015): "Asset Pricing with Dynamically Inconsistent Agents," *Working Paper, Stockholm School of Economics*.

Kihlstrom, R. E., and L. J. Mirman (1974): "Risk Aversion with Many Commodities," *Journal of Economic Theory*, 8, 361–388.

Köszegi, B., and M. Rabin (2007): "Reference-Dependent Risk Attitudes," *American Economic Review*, 97(4), 1047–1073.

——— (2009): "Reference-Dependent Consumption Plans," *American Economic Review*, 99(3), 909–936.

Kreps, D., and E. Porteus (1978): "Temporal Resolution of Uncertainty and Dynamic Choice Theory," *Econometrica*, 46(1), 185–200.

Kunreuther, H. (1984): "Causes of Underinsurance against Natural Disasters," *Geneva Papers on Risk and Insurance*, 9(31), 206–220.

Laibson, D. (1997): "Golden Eggs and Hyperbolic Discounting," *Quarterly Journal of Economics*, 112(2), 443–477.

Loewenstein, G. F., and D. Prelec (1992): "Anomalies in Intertemporal Choice: Evidence and an Interpretation," *Quarterly Journal of Economics*, 107(2), 573–597.

Loewenstein, G. F., and R. Thaler (1989): "Anomalies: Intertemporal Choice," *Journal of Economic Perspectives*, 3(4), 181–193.

Loomes, G. (2010): "Modelling Choice and Valuation in Decision Experiments," *Psychological Review*, 117(3), 902–924.

Loomes, G., and R. Sugden (1986): "Disappointment and Dynamic Consistency in Choice under Uncertainty," *Review of Economic Studies*, 53(2), 271–282.

——— (1987): "Testing for Regret and Disappointment in Choice Under Uncertainty," *Economic Journal*, 97, 118–129.

Lovallo, D., and D. Kahneman (2000): "Living with Uncertainty: Attractiveness and Resolution Timing," *Journal of Behavioral Decision Making*, 13, 179–190.

Luce, D. R. (2001): "Reduction Invariance and Prelec's Weighting Functions," *Journal of Mathematical Psychology*, 45, 167–179.

Luce, R. D., B. A. Mellers, and S.-J. Chang (1993): "Is Choice the Correct Primitive? On Using Certainty Equivalents and Reference Levels to Predict Choices Among Gambles," *Journal of Risk and Uncertainty*, 6, 115–143.

Machina, M. (2009): "Risk, Ambiguity, and the Rank-Dependence Axioms," *American Economic Review*, 99, 385–392.

——— (2014): "Ambiguity Aversion with Three or More Outcomes," *American Economic Review*, 104(12), 3814–3840.

Masatlioglu, Y., and C. Raymond (2016): "A Behavioral Analysis of Stochastic Reference Dependence," *American Economic Review*, 106(9), 2760–2782.

Mazur, J. E. (1987): "An Adjustment Procedure for Studying Delayed Reinforcement," *The Effect of Delay and Intervening Events on Reinforcement Value. Commons, M.L., Mazur, E.M. and Rachlin, H. (Eds.), Hillsdale, NJ: Erbaum*.

Meade (1957): "How Secure is Air Trip Insurance?," *Virginia State Bar Association Reports*, 68, 215–221.

Nebout, A., and D. Dubois (2014): "When Allais Meets Ulysses: Dynamic Axioms and the Common Ratio Effect," *Journal of Risk and Uncertainty*, 48(1), 19–49.

Noussair, C., and P. Wu (2006): "Risk Tolerance in the Present and the Future: An Experimental Study," *Managerial and Decision Economics*, 27, 401–412.

O'Donogue, T., and J. Somerville (2018): "Modeling Risk Aversion in Economics," *Journal of Economic Perspectives*, 32.

Öncüler, A., and S. Onay (2009): "How Do We Evaluate Future Gambles? Experimental Evidence on Path Dependency in Risky Intertemporal Choice," *Journal of Behavioral Decision Making*, 22, 280–300.

Palacios-Huerta, I. (1999): "The Aversion to the Sequential Resolution of Uncertainty," *Journal of Risk and Uncertainty*, 18, 249–269.

Polkovnichenko, V., and F. Zhao (2013): "Probability Weighting Functions Implied in Options Prices," *Journal of Financial Economics*, 107, 580–609.

Prelec, D. (1998): "The Probability Weighting Function," *Econometrica*, 66(3), 497–527.

——— (2004): "Decreasing Impatience: A Criterion for Non-stationary Time Preference and "Hyperbolic" Discounting," *Scandinavian Journal of Economics*, 106(3), 511–532.

Prelec, D., and G. F. Loewenstein (1991): "Decision Making over Time and under Uncertainty: A Common Approach," *Management Science*, 37(7), 770–786.

Preston, M. G., and P. Baratta (1948): "An Experimental Study of the Auction-Value of an Uncertain Outcome," *American Journal of Psychology*.

Quiggin, J. (1982): "A Theory of Anticipated Utility," *Journal of Economic Behavior and Organization*, 3, 323–343.

Quiggin, J., and J. Horowitz (1995): "Time and Risk," *Journal of Risk and Uncertainty*, 10, 37–55.

Rachlin, H., A. Raineri, and D. Cross (1991): "Subjective Probability and Delay," *Journal of the Experimental Analysis of Behavior*, 55(2), 233–244.

Read, D. (2001): "Is Time-Discounting Hyperbolic or Subadditive?," *Journal of Risk and Uncertainty*, 23(1), 5–32.

Read, D., and P. Roelofsma (2003): "Subadditive versus Hyperbolic Discounting: A Comparison of Choice and Matching," *Organizational Behavior and Human Decision Processes*, 91, 140–153.

Richard, S. F. (1975): "Multivariate Risk Aversion, Utility Independence and Separable Utility Functions," *Management Science*, 22(1), 12–21.

Rieger, M., and M. Wang (2006): "Cumulative Prospect Theory and the St. Petersburg Paradox," *Economic Theory*, 28, 665–679.

Rohde, K. I. M. (2018): "Measuring Decreasing and Increasing Impatience," *Management Science*, https://doi.org/10.1287/mnsc.2017.3015.

Rottenstreich, Y., and C. K. Hsee (2001): "Money, Kisses and Electric Shocks: On the Affective Psychology of Risk," *Psychological Science*, 12(3), 185–190.

Rubinstein, A. (1988): "Similarity and Decision-Making Under Risk (Is There a Utility Theory Resolution to the Allais Paradox?)," *Journal of Economic Theory*, 46, 145–153.

Safra, Z., and U. Segal (1998): "Constant Risk Aversion," *Journal of Economic Theory*, 83, 19–42.

Sagristano, M., Y. Trope, and N. Liberman (2002): "Time-dependent Gambling: Odds Now, Money Later," *Journal of Experimental Psychology*, 131(3), 364–376.

Saito, K. (2011): "Strotz Meets Allais: Diminishing Impatience and the Certainty Effect: Comment," *American Economic Review*, 101(5), 2271–2275.

Saito, K. (2015): "A Relationship between Risk and Time Preferences," *mimeo*.

Segal, U. (1987a): "Some Remarks on Quiggin's Anticipated Utility," *Journal of Economic Behavior and Organization*, 8, 145–154.

——— (1987b): "The Ellsberg Paradox and Risk Aversion: An Anticipated Utility Approach," *International Economic Review*, 28(1), 175–202.

——— (1990): "Two-Stage Lotteries Without the Reduction Axiom," *Econometrica*, 58(2), 349–377.

Shelley, M. (1994): "Gain/Loss Asymmetry in Risky Intertemporal Choice," *Organizational Behavior and Human Decision Processes*, 59, 124–159.

Spalt, O. G. (2013): "Probability Weighting and Employee Stock Options," *Journal of Financial and Quantitative Analysis*, 48(4), 1085–1118.

Starmer, C. (2000): "Developments in Non-Expected Utility Theory: The Hunt for a Descriptive Theory of Choice under Risk," *Journal of Economic Literature*, 38(2), 332–382.

Starmer, C., and R. Sugden (1989): "Violations of the Independence Axiom in Common Ratio Problems: An Experimental Test of Some Competing Hypotheses," *Annals of Operations Research*, 19, 79–102.

Stevenson, M. (1992): "The Impact of Temporal Context and Risk on the Judged Value of Future Outcomes," *Organizational Behavior and Human Decision Processes*, 52, 455–491.

Sugden, R. (2004): "Alternatives to Expected Utility: Foundations," in *Handbook of Utility Theory: Volume II: Extensions*, pp. 685–755. Kluwer, Dordrecht.

Sydnor, J. (2010): "(Over)insuring Modest Risks," *American Economic Journal: Applied Economics*.

Thaler, R. (1981): "Some Empirical Evidence on Dynamic Inconsistency," *Economic Letters*, 8, 201–207.

Thaler, R. H., A. Tversky, D. Kahneman, and A. Schwartz (1997): "The Effect of Myopia and Loss Aversion on Risk Taking: An Experimental Test," *Quarterly Journal of Economics*, 112(2), 647–661.

Tversky, A., and D. Kahneman (1992): "Advances in Prospect Theory: Cumulative Representation of Uncertainty," *Journal of Risk and Uncertainty*, 5, 297–323.

van Binsbergen, J., M. Brandt, and R. Koijen (2012): "On the Timing and Pricing of Dividends," *American Economic Review*, 102(4), 1596–1618.

van de Kuilen, G., and P. Wakker (2011): "The Midweight Method to Measure Attitudes Toward Risk and Ambiguity," *Management Science*, 57(3), 582–598.

Viscusi, W. K. (2010): "The Hold-up Problem. why it is Urgent to Rethink the Economics of Disaster Insurance Protection," *in: Erwann, M.-K. and Slovic, P. (Eds.), The Irrational Economist: Making Decisions in a Dangerous World*, pp. 142–148.

von Gaudecker, H.-M., A. van Soest, and E. Wengström (2011): "Heterogeneity in Risky Choice Behaviour in a Broad Population," *American Economic Review*, 101(2), 664–694.

von Peter, G., S. von Dahlen, and S. Saxena (2012): "Unmitigated Disasters? New Evidence on the Macroeconomic Cost of Natural Catastrophies," *Bank of International Settlements Working Paper*, 394.

Wakker, P. (2010): *Prospect Theory for Risk and Ambiguity*. Cambridge University Press, Cambridge.

Walther, H. (2003): "Normal-Randomness Expected Utility, Time Preference and Emotional Distortions," *Journal of Economic Behavior and Organization*, 52, 253–266.

Weber, B., and G. Chapman (2005): "The Combined Effects of Risk and Time on Choice: Does Uncertainty Eliminate the Immediacy Effect? Does Delay Eliminate the Certainty Effect?," *Organizational Behavior and Human Decision Processes*, 96, 104–118.

Wu, G. (1999): "Anxiety and Decision Making with Delayed Resolution of Uncertainty," *Theory and Decision*, 46, 159–198.

Yaari, M. (1987): "The Dual Theory of Choice under Risk," *Econometrica*, 55(1), 95–115.