THE MISSING LINK: Unifying Risk Taking and Time Discounting*

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Abstract

Almost all important decisions in people's lives entail risky and delayed consequences. Regardless of whether we make choices involving health, wealth, love or education, almost every choice involves uncertain costs and benefits that take time to materialize. Because risk and delay often arise simultaneously, theories of decision making should be capable of explaining behavior when both risk and and delay are present. There is, in fact, a growing body of evidence that risk tolerance and patience interact in important ways. Here we show that the risk of survival of future prospects conjointly with people's proneness to Allais-type behavior generates a unifying framework for explaining time-dependent risk taking, risk-dependent time discounting, hyperbolic time discounting, preferences for late resolution of uncertainty, the co-existence of increasing relative risk aversion and the magnitude effect in time discounting, as well as other puzzling interaction effects between risk and time.

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1 Introduction

Whatever the nature of our decisions, may they concern health, wealth, love or education, hardly ever can we be sure about their outcomes. Thus, it is an important task for economists to understand, model and predict decisions under risk. However, it seems difficult to paint a coherent picture of people's risk preferences because in some situations people appear to be extremely risk averse while in others they appear to be extremely risk tolerant. For example, many consumers purchase warranties for household appliances at exorbitant prices, i.e. they display extreme risk aversion in a situation that involves comparatively low stakes (Cicchetti and Dubin, 1994). According to the standard workhorse of economics, expected utility theory, consumers should be approximately risk neutral in this case (Loomes and Segal, 1994). On the other hand, many are reluctant to buy adequate health insurance unless forced to do so by law, even though they will agree that health is the most valuable good in their lives (Schoen, Hayes, Collins, Lippa, and Radley, 2014). Similarly, stock market investors appear to be overly risk averse (Mehra and Prescott, 2003) whereas many people are not willing to take out highly subsidized insurance for natural disasters even though not only their wealth but also their lives are at stake (Viscusi, 2010).

In this paper we argue that these contradictory findings can be explained by adequately accounting for the role of time. Time is an important component of the majority of decisions because their outcomes are not only uncertain but also take time to materialize. Mounting experimental evidence, summarized below, suggests that the passage of time is not an independent dimension of risky prospects but rather interacts with risk in complex ways. In particular, there are two aspects that decision makers care about: first, the *timing of uncertainty resolution*, i.e. when it becomes known which one of the future consequences will actually obtain, and second, the *process of uncertainty resolution*, i.e. whether uncertainty resolves in one shot or sequentially over time. As we will show, these features of uncertainty resolution have significant implications for behavior, rendering the decision maker more or less risk averse depending on the circumstances. Therefore, understanding the experimental data is tantamount to understanding real-world behavior.

Table 1 presents nine facts on interactions between risk and time, organized along six dimensions, which we discuss in detail in Section 2. First, revealed risk tolerance and revealed patience increase with time delay. Second, risk taking and time discounting behavior are process dependent, i.e. it makes a difference whether a future prospect is evaluated in one shot or sequentially over the course of time. One-shot prospect values are generally higher than values obtained by a sequential evaluation procedure. This first set of findings relates to real-world behavior in the following way. For example, as the expected life time of household appliances is comparatively short, risk aversion is relatively high and it is likely that appliances will be insured. On the other hand, stock investments usually have a long time horizon, which should, in principle, make decision makers comparatively risk tolerant. However, information on stock prices can be obtained practically continuously, such that uncertainty concerning portfolio value is perceived to resolve sequentially over the course of time, which has a dampening effect on risk tolerance. Health, on the contrary, usually deteriorates slowly and for a long time imperceptibly so, such that uncertainty is perceived to resolve at some indeterminate time in the distant future, which increases risk tolerance and decreases the propensity to take out insurance.

The list of observed interaction effects is much longer, however. The third dimension concerns the timing of uncertainty resolution, with late resolution of uncertainty often preferred to early resolution. Fourth, the presence of risk influences time discounting in an unexpected way: Certain outcomes seem to be discounted more heavily than uncertain ones. Fifth, people's evaluations of future risky payoffs depend on the order by which they are discounted for risk and for time. Finally, it is a well-known fact that relative risk aversion increases with stake size. However, this characteristic of risk preferences is difficult to reconcile with the magnitude effect in discounting according to which patience increases with stake size. The co-existence of both effects requires that the elasticity of the utility function increases with stake size in the time domain and decreases with stake size in the risk domain. These conditions cannot hold simultaneously if the utility function is the sole carrier of preferences over monetary outcomes.

Table 1: Nine Facts on Risk Taking and Time Discounting

Dimension	Fact	Revealed risk tolerance	Fact	Revealed patience
Delay dependence	#1	increases with delay	#2	increases with delay
Process dependence	#3	higher for one-shot valuation	#4	higher for one-shot valuation
Timing dependence	#5	higher for late uncertainty resolution		–
Risk dependence		–	#6	higher for risky payoffs
Order dependence	#7	depends on order of delay and risk discounting		–
Stake dependence	#8	lower for higher stakes	#9	higher for higher stakes

The table describes nine facts regarding the effects of delay, process, timing, risk and sequence on risk taking and discounting behavior.

In the following we show that all these effects can indeed be rationalized within a unifying framework that relies on two basic ideas. First, there is risk attached to any future prospect as only immediate consequences can be totally certain. If future prospects are inherently risky, people's risk tolerance must play a role in the valuation of future prospects. Therefore, the second pillar of our model pertains to the characteristics of (atemporal) risk preferences. There is abundant evidence from the field and the laboratory that risk taking behavior depends nonlinearly on the objective probabilities (for a recent review see Fehr-Duda and Epper (2012)). For example, Barseghyan, Molinari, O'Donoghue, and Teitelbaum (2013) show that one of the most important drivers of people's deductible choices is the overweighting of small probabilities. In the laboratory, models involving probability weighting, such as rank dependent utility (Quiggin, 1982), cumulative prospect theory (Tversky and Kahneman, 1992) and more recently salience theory (Bordalo, Gennaioli, and Shleifer, 2012), have proven to be the most successful contenders of expected utility theory.

In our approach, we rely on a specific characteristic of probability weighting, proneness to Allais-type *common-ratio violations* that is one of the most widely replicated experimental regularities in the lab, found both in human and animal behavior: Scaling down the probabilities of the non-zero outcomes in a pair of prospects frequently leads to preference reversals (Allais, 1953; Hagen, 1972; Kahneman and Tversky, 1979; MacCrimmon and Larsson, 1979; Battalio, Kagel, and MacDonald, 1985; Kagel, MacDonald, and Battalio, 1990; Chark, Chew, and Zhong, 2014; Nebout and Dubois, 2014; Burghart, Epper, and Fehr, 2014). Here we show that this feature of probability weighting conjointly with inherent future risk provides an integrative account of all the facts #1 to #7. Furthermore, increasing atemporal relative risk aversion implies the magnitude effect in discounting, thereby harmonizing facts #8 and #9. Thus, our key assumptions provide a unifying account for all the facts listed in Table 1 and a rationale for many real-world phenomena that have been puzzling economists for a long time.

We are not the first to note that *"[a]nything that is delayed is almost by definition uncertain"* (Prelec and Loewenstein (1991), p.784). Several authors have abandoned the separability of risk and time as well to develop more realistic models. However, they usually focus on a single anomaly and have not attempted to attribute the entirety of the observed interaction effects to a single driving force. To the best of our knowledge, no satisfactory account for the coexistence of the magnitude effects in risk taking and time discounting has been provided so far, either.

Several papers identified uncertainty as a potential cause of hyperbolic discounting (Sozou, 1998; Dasgupta and Maskin, 2005; Bommier, 2006; Pennesi, 2014). Most closely related to our research are the models introduced by Halevy (2008) and Walther (2010) that derive hyperbolic discounting from nonlinear probability weighting. Walther's approach is based on his model of affective utility (Walther, 2003) that endogenously generates an inverse S-shaped probability weighting function and, consequently, a short-run hyperbolic decline of the implied discount function. However, this discount function turns out to be U-shaped as discount rates start to increase again at some point in time. Halevy's discount function does not exhibit this pattern because it is derived from subproportional probability weights,[1] whereas Walther's probability

[1]Subproportionality captures the common-ratio property and is formally defined in Section 4. Halevy (2008) bases his analysis on probability weighting functions of increasing elasticity, which is equivalent to subproportionality, but he generally focuses on convex transformations.

weighting function does not exhibit global subproportionality. Neither author addresses delay dependence of risk tolerance nor process, timing, and order dependence.

Survival risk does not play a role in Baucells and Heukamp (2012)'s axiomatic model for the limited domain of prospects with one non-zero outcome which captures interactions of risk tolerance and time discounting behavior by a psychological distance function. The psychological distance of an outcome is assumed to increase with increasing delay or decreasing probability. Similarly to ours, their model is inspired by the common-ratio effect in risk taking behavior but, contrary to our approach, it is a descriptive representation of behavior rather than a structural approach. In their setting, increasing risk tolerance and increasing patience are equivalent. Hence, hyperbolic discounting is built into the assumptions and not derived from probability weighting, in contrast to Halevy (2008) and Walther (2010).[2] Moreover, they do not deal with the issues of process, timing, and order dependence.

Process dependence is the focus of Palacios-Huerta (1999)'s contribution. He shows that, in the context of Gul (1991)'s model, a disappointment averse decision maker exhibits much larger risk aversion when she evaluates a multi-stage prospect sequentially rather than in one shot. Dillenberger (2010) provides an axiomatic underpinning for this result. He proves that, under the assumption of recursive valuation of multi-stage prospects, a weak preference for one-shot resolution of uncertainty is equivalent to risk preferences satisfying a novel axiom, negative certainty independence. This axiom weakens the standard independence axiom and allows for common-ratio violations but is silent on their actual occurrence (Cerreia-Vioglio, Dillenberger, and Ortoleva, forthcoming). Dillenberger (2010) also provides an insightful discussion of the consequences of a preference for one-shot resolution of uncertainty on the value of information.

Finally, our discussion of the relationship between subproportional probability weighting and one-shot resolution of uncertainty is based on the seminal work by Segal (1987a,b, 1990) who analyzes the evaluation of multi-stage prospects in the domain of rank-dependent utility. Since these papers deal with dynamic, but essentially atemporal, situations, they are not concerned with time discounting or the other facts listed in Table 1.

The remainder of the paper is organized as follows: Section 2 discusses the interaction effects identified by experimental work in detail and argues that none of the existing explanations is able to accommodate all of them. The key assumptions of our model and their implications are discussed in sections 3 and 4. Model predictions and the corresponding experimental evidence are summarized in section 5. Finally, section 6 concludes. Proofs and supplementary materials are available in the online appendix.

[2] Baucells and Heukamp (2012) claim that their model can explain the experimental results in Abdellaoui, Diecidue, and Öncüler (2011) who find that the probability weighting function is more elevated for longer delays t. However, Baucells and Heukamp (2012)'s formulation of delay-dependent probability weights $w_t(p) = w(pe^{-r_x t})$ with probability p and discount rate r_x appears to imply the opposite (p. 836).

2 Nine Facts on Risk Taking and Time Discounting

Turning to the first phenomenon, delay dependence of risk taking behavior, Table 1 indicates that risk tolerance appears not to be a stable characteristic of people's behavior. Rather, it is higher for payoffs materializing in the future than for payoffs materializing in the present (Shelley, 1994; Ahlbrecht and Weber, 1997; Sagristano, Trope, and Liberman, 2002; Noussair and Wu, 2006; Coble and Lusk, 2010; Abdellaoui, Diecidue, and Öncüler, 2011). This finding may have far-reaching implications: If people are willing to tolerate high risks for prospects in the remote future, they may be reluctant to support policies combating global warming or to buy insurance for natural hazards. Greater risk tolerance for future payoffs goes against the grain of the most widely used models of risk taking behavior, such as expected utility theory and its prominent alternatives rank-dependent utility, cumulative prospect theory, and theories of disappointment aversion.

It is well known by now that delay dependence is also manifest in discounting behavior, which constitutes empirical fact #2. Contrary to the prediction of standard discounted utility theory, people behave more patiently with respect to more remote payoffs, which implies that people's discount rates are not constant but decline with the length of delay (Strotz, 1955; Benzion, Rapoport, and Yagil, 1989; Loewenstein and Thaler, 1989; Ainslie, 1991; Loewenstein and Prelec, 1992; Halevy, forthcoming). This regularity has triggered a large literature on so-called hyperbolic preferences (e.g. Laibson (1997); Frederick, Loewenstein, and O'Donoghue (2002)). Hyperbolic discounting has been readily adopted by applied economics in many fields such as saving behavior, procrastination, addiction, and retirement decisions. Hyperbolic preference models provide, however, no explanation for interactions between time and risk.

Another regularity in the data concerns facts #3 and #4, the process dependence of risk taking and time discounting behavior. It generally makes a difference whether future prospects are assessed in one shot or frequently over the course of time. In the domain of risk, people tend to invest less conservatively, i.e. they take on more risk, when they are informed about the outcomes of their decisions only at the end of the investment period rather than intermittently (Gneezy and Potters, 1997; Thaler, Tversky, Kahneman, and Schwartz, 1997; Bellemare, Krause, Kröger, and Zhang, 2005; Gneezy, Kapteyn, and Potters, 2003; Haigh and List, 2005). Stock market data is easily accessible and thus provides continuous feedback on portfolio performance. If people watch closely how uncertainty resolves, their risk tolerance may be much lower than when they have no access to this information. Hence, frequent portfolio evaluation may be an important factor driving the large equity premium, i.e. the return earned by stocks in excess of that earned by relatively risk-free government bonds, as it has been observed in the U.S. and in other industrialized countries (Mehra, 2006). The magnitude of the observed equity premium has been puzzling economists for more than 25 years because it is hard to reconcile with plausible levels of risk aversion when interpreted within the framework of expected utility theory. Some authors have attributed the aversion to frequent information and, consequently the large equity premium (Benartzi and Thaler, 1995; Barberis, Huang, and Santos, 2001), to loss aversion, an integral concept of prospect theory (Tversky and Kahneman, 1992), that has increasingly attracted economists' attention in the last decade (Gächter, Johnson, and Herrmann, 2007; Köszegi and Rabin, 2007; Abeler, Falk, Goette, and Huffman, 2011). While loss aversion may explain why people are more risk averse when assessing portfolio performance frequently it cannot account for the other findings on observed risk taking and discounting behavior.

In the domain of time discounting, a similar phenomenon of process dependence has been observed, listed as fact #4 in Table 1: The discounting shown over a particular delay is greater when the delay is divided into subintervals than when it is left undivided (Read, 2001; Read and Roelofsma, 2003; Ebert and Prelec, 2007; Epper, Fehr-Duda, and Bruhin, 2009; Dohmen, Falk, Huffman, and Sunde, 2012). For example, discounting over a one-year period will be greater when the year is divided into two subperiods of six months, and even more so when it is divided into subperiods of, say, twelve months. This regularity has been labeled *subadditive discounting*. So evaluating a future payoff sequentially rather than in one shot typically decreases its value - decision makers exhibit less patience in this case, an effect equivalent to the process dependence of risk tolerance.

The third row in Table 1 refers to fact #5, the effect of the timing of uncertainty resolution on risk taking behavior. Principally, knowing the outcome of one's decision before the actual payment date should be beneficial because one can integrate this information into one's future plans. Therefore, according to the standard model of risky choice, information is valuable if it enables the decision maker to choose better strategies. If she does not or cannot condition her actions on what she learns she should be indifferent toward the timing of uncertainty resolution, i.e. information about realized outcomes should be worthless to her (Grant, Kajii, and Polak, 1998). Hence, the value of information should be nonnegative. In contrast to this prediction, many people prefer uncertainty to resolve later rather than sooner (Chew and Ho, 1994; Ahlbrecht and Weber, 1996; Arai, 1997; Lovallo and Kahneman, 2000; Eliaz and Schotter, 2007; von Gaudecker, van Soest, and Wengström, 2011). For instance, some people with a family history of a genetic disorder may choose not to be informed whether they are affected by the disorder or not. Such an intrinsic preference for resolution timing cannot be accommodated by the standard theory of risk taking but is modeled by an additional preference parameter (Kreps and Porteus, 1978; Chew and Epstein, 1989; Grant, Kajii, and Polak, 2000).

Fact #6 pertains to a number of experimental studies that report systematic effects of risk on discounting behavior: Discount rates for certain future payoffs tend to be higher than discount rates for risky future payoffs (Stevenson, 1992; Ahlbrecht and Weber, 1997). Higher discount rates for certain payoffs not only run counter to intuition but also contradict the standard model of discounting according to which the same level of patience applies to risky and certain future prospects. Risk-dependent discounting is also evident in *diminishing immediacy*: People's preference for present certain outcomes over delayed ones, immediacy, weakens drastically when the outcomes become risky: Whereas many people prefer a smaller immediate reward to a larger delayed one, merely a minority continue to do so when both rewards are made probabilistic - they behave as if they discounted the risky reward less heavily than the original certain one (Keren and Roelofsma, 1995; Weber and Chapman, 2005).

Furthermore, the valuation of future prospects appears to be *order dependent*, labeled fact #7: It makes a difference whether a risky future payoff is first devalued for risk and then for delay or in the opposite order (Öncüler and Onay, 2009). When payoffs are discounted for risk first they are assigned a less favorable value than in the reverse case. Moreover, the delay-first value practically coincides with the value reported when both dimensions are accounted for in one single operation. This finding implies that risk tolerance revealed for future prospects systematically depends on the order in which discounting for risk and for time is performed.

Finally, both risk taking and time discounting depend on stake size, facts #8 and #9 (Binswanger (1981); Holt and Laury (2002); Fehr-Duda, Bruhin, and Epper (2010); Frederick, Loewenstein, and O'Donoghue (2002)). That relative risk aversion increases with stake size does not contradict expected utility theory, which is silent on the effect of outcome magnitude on the curvature of the utility function. Stake-dependence of discounting, however, is bad news for standard discounted utility theory, according to which discount weights are independent of outcome magnitude. More importantly, however, increasing relative risk aversion and increasing impatience are mutually exclusive characteristics if the same utility function is assumed to drive behavior in both decision domains.

The evidence discussed above demonstrates striking parallels between the susceptibility of observed risk tolerance and observed patience to the length of delay as well as the evaluation process: Deferring payoffs to the remote future apparently makes people both more risk tolerant and more patient, and one-shot evaluation has favorable effects on risk taking as well as on time discounting behavior. Since the effects appear not to be arbitrary aberrations from the predictions of the existing models, one might ask whether there is a common mechanism governing observed behavior that accounts for delay, process, timing, risk, and order dependence. The stake dependence of risk taking and time discounting follows a different logic and seems, prima vista, hard to integrate.

3 Key Assumptions

Our model builds on two basic ideas: First, there is risk attached to any future prospect. This risk inherent in the future, *survival risk* for short, may stem from different sources. At the personal level, it refers to a general feeling of "something may go wrong" due to unexpected contingencies, such as a check getting lost in the mail. Another important channel through which survival risk may manifest itself is the institutional environment. Environments where property rights are only weakly protected or institutions of contract enforcement are not reliable, as is the case in many developing countries, are characterized by high survival risk. This (uninsurable) risk turns allegedly guaranteed payoffs into risky ones and introduces an additional layer of risk over and above the objective probability distributions of risky payoffs (henceforth referred to as *base risk*). Consequently, there are two distinct types of risk, time-independent *base risk* and time-dependent *survival risk*.

The second pillar of our model pertains to the characteristics of risk preferences. Abundant empirical evidence has demonstrated that risk taking behavior depends nonlinearly on the probabilities, which is inconsistent with expected utility theory. Inspired by one of Allais (1953)'s famous examples, Kahneman and Tversky (1979) presented subjects with the following decision situation: Subjects had to choose between 3000 dollars for certain and 4000 dollars materializing with a probability of 80%. Most people chose the certain option of 3000 dollars. When confronted with the choice between a 25%-chance of receiving 3000 dollars and a 20%-chance of receiving 4000 dollars, the majority opted for the 4000-dollar alternative. Scaling down the probabilities of 100% and 80% by a common factor, in this example $1/4$, induced many people to reverse their preferences.[3]

Such Allais-type behavior is inconsistent with expected utility theory but consistent with a large number of alternative models (for a review see Starmer (2000)). In this paper, we rely on rank-dependent utility theory (RDU) (Quiggin, 1982), which represents common-ratio violations directly by a nonlinear transformation of the probabilities. By convention, the probability weighting function maps the weight attached to the probability of a prospect's best outcome. RDU has several attractive features. First, RDU respects completeness, transitivity, continuity, and first-order stochastic dominance, qualities that many economists are hesitant to dispense with. Second, the common inverse S-shape of the probability weighting function generates overweighting of a prospect's extreme outcomes and underweighting of its intermediate outcomes, which nicely captures the notion that more extreme outcomes within a given prospect are more salient. Finally, RDU displays first-order attitudes toward risk, i.e. preferences between prospects whose consequences are sufficiently close to one another do not necessarily tend to risk neutrality. In this sense, experimental evidence favors rank-dependent utility theory over many other non-expected-utility approaches that can accommodate the common-ratio effect but only permit second-order risk aversion (Sugden, 2004).

A recent experimental paper on risky delayed prospects contested the usefulness of RDU, however. In Andreoni and Sprenger (2012)'s experiment, subjects allocate a given budget to two

[3] This example constitutes a special case of common-ratio violations, known as *certainty effect*, as the smaller outcome in the first decision situation, 3000 dollars, materializes with certainty. There is also ample evidence for general common-ratio violations that do not involve a certain outcome (Kahneman and Tversky, 1979; MacCrimmon and Larsson, 1979; Loomes and Sugden, 1987; Nebout and Dubois, 2014; Chark, Chew, and Zhong, 2014).

different future payment dates. When these future payments are risky, subject tend to choose a more evenly balanced portfolio than in the case of guaranteed payments, i.e. they exhibit intertemporal risk aversion. Andreoni and Sprenger (2012) claim that their experimental data cannot be explained by models involving probability weighting. Epper and Fehr-Duda (forthcoming) demonstrate, however, that all the key findings in Andreoni and Sprenger (2012) can be accommodated by RDU if subjects care about portfolio risk rather than about the risks of the delayed payments in isolation. The portfolio view is particularly plausible in a setting where a budget has to be allocated to different future payment dates. Epper and Fehr-Duda (forthcoming) argue that RDU not only accommodates intertemporal risk aversion but also provides the most convincing account of the Andreoni and Sprenger (2012) data, compared with other candidate approaches. Furthermore, RDU can handle correlation aversion as well: When decision makers evaluate risky consumption streams they often have a preference for diversifying consumption across time, i.e. they prefer some good and some bad to all or nothing (Kihlstrom and Mirman, 1974; Richard, 1975; Epstein and Tanny, 1980; Bommier, 2007; Denuit, Eeckhoudt, and Rey, 2010). Epper and Fehr-Duda (forthcoming) show that RDU implies correlation aversion if the decision maker is sufficiently pessimistic.

An intuitive explanation for common-ratio violations is the fear of disappointment: Losing a gamble over very likely 4000 dollars is anticipated to be much more disappointing than losing a gamble over 4000 dollars that have only a small chance of materializing in the first place. On the other hand, winning 4000 dollars in the unlikely situation of a 20%-chance may trigger feelings of elation. Thus, if people are prone to disappointment and elation, their behavior appears to depend nonlinearly on the probabilities (Bell, 1985; Loomes and Sugden, 1986; Gul, 1991; Wu, 1999; Walther, 2003).[4]

Another potential source of probability weighting is loss aversion, a concept introduced by Tversky and Kahneman (1992) and since then readily adopted by many researchers. Tversky and Kahneman (1992) provided evidence for their claim that preferences are reference dependent and that losses with respect to the reference point loom larger than gains. The main weakness of their approach, however, is its failure to provide a convincing account of reference-point formation. In order to address this problem Köszegi and Rabin (2007) developed a model that endogenizes the reference point. Masatlioglu and Raymond (2014) show that Köszegi and Rabin (2007)'s choice-acclimating personal equilibrium with linear gain-loss utility is equivalent to rank-dependent utility with a specific convex probability transformation. In that sense, probability weighting

[4]Perceptual and procedural factors are potential drivers of probability distortions as well. The fathers of prospect theory, Kahneman and Tversky, attributed probability dependence to the psychophysics of perception according to which the sensitivity toward changes in probabilities diminishes with the distance to the natural reference points of certainty and impossibility (Tversky and Kahneman, 1992). Several other contributions focused on procedural aspects of choice (Rubinstein, 1988; Loomes, 2010). In these models, a prospect's value depends not only on the prospect's own characteristics but also on other prospects in the choice set. A recent contribution in this category is Bordalo, Gennaioli, and Shleifer (2012) who posit that probabilities are distorted in favor of payoffs that are perceived as particularly salient.

in a rank-dependent model can be interpreted as a reduced form generated by psychological mechanisms such as loss aversion or anticipated emotions.

Experimental estimates of average probability weights typically yield inverse S-shaped probability weighting curves, underweighting large probabilities and overweighting small probabilities of the best outcome, which is also a common pattern in individual data (Gonzalez and Wu, 1999; Bruhin, Fehr-Duda, and Epper, 2010). Aside from inverse-S shapes, convex weighting curves, globally underweighting probabilities, comprise another common category of shapes (van de Kuilen and Wakker, 2011). Since we are primarily concerned with common-ratio violations, we need to put more structure on the probability weighting function. Common-ratio violations are mapped by a specific characteristic of probability weights, subproportionality. In principle, subproportionality can be exhibited by both inverse-S shapes (e.g. Prelec (1998)) and convex shapes of probability weighting curves (e.g. Gul (1991) or Masatlioglu and Raymond (2014)).

4 The Model

We consider the set of binary prospects $P = (x_1, p; x_2)$ with payoffs $x_1 > x_2 \geq 0$, probability p of the larger payoff x_1 and probability $1 - p$ of the smaller payoff x_2.[5] We assume that a decision maker's true atemporal risk preferences over such prospects, played out and paid out instantaneously, can be represented by a rank-dependent functional:

$$\begin{aligned} V(P) &= u(x_1)w(p) + u(x_2)(1 - w(p)) \\ &= [u(x_1) - u(x_2)]\, w(p) + u(x_2) \end{aligned} \tag{1}$$

where u measures the utility of monetary amounts x, and w denotes the subjective probability weight attached to p, the probability of the better outcome x_1. As usual, both u and w are assumed to be monotonically increasing, w to be twice differentiable and to satisfy $w(0) = 0$ and $w(1) = 1$. A summary of the model variables is provided in Table 2. Technically, common-ratio violations are represented by *subproportionality* of the probability weighting function. Subproportionality decreases the decision maker's sensitivity to disappointment for scaled-down probabilities, i.e. risky outcomes are potentially more disappointing the higher is their ex-ante probability of materializing. In this sense, the loss of certainty hurts more than the scaling down of a probability less than one does.

Formally, subproportionality is defined as follows (Prelec, 1998): Subproportionality holds if

[5] Our approach can be easily generalized to $n > 2$ outcomes provided that survival risk does not change the rank order of the prospects, i.e. if "something may go wrong" is encoded as an outcome no better than the prospects' minimum outcome.

$1 \geq p > q > 0$ and $0 < \lambda < 1$ imply the inequality

$$\frac{w(p)}{w(q)} > \frac{w(\lambda p)}{w(\lambda q)}. \tag{2}$$

Subproportionality implies the certainty effect, which constitutes the special case of $p = 1$. Therefore, $w(\lambda q) > w(\lambda)w(q)$ is satisfied for any λ, q such that $0 < \lambda, q < 1$. Many functional specifications proposed in the literature exhibit subproportionality over some probability range under appropriate parameter restrictions (see Appendix C and section 3.6 in Fehr-Duda and Epper (2012) for a review). Perhaps the most prominent representative of a globally subproportional function is Prelec (1998)'s flexible two-parameter specification, designed to map common-ratio violations. Gul (1991)'s theory of disappointment aversion, for example, implies a strictly convex subproportional function in the context of two-outcome prospects. Another interesting specimen is the probability weighting function discussed in Delquié and Cillo (2006). In the context of RDU, their model of disappointment aversion generates a subproportional second-order polynomial that is equivalent to the one implied by Köszegi and Rabin (2007)'s choice-acclimating personal equilibrium, which provides an endogenous reference point (Masatlioglu and Raymond, 2014).[6] This concept captures the idea that a decision maker commits to a choice long before uncertainty is resolved, and is, therefore, particularly plausible in the context of our model. Bordalo, Gennaioli, and Shleifer (2012) derive (discontinuous) context-dependent probability distortions from their salience theory. While their concave segment is superproportional, the convex segment is equivalent to a subproportional probability weighting function of the Rachlin, Raineri, and Cross (1991) variety. The psychological mechanisms underlying probability weighting, therefore, often imply subproportionality. In Appendix C we provide a discussion of global and partial subproportionality.

If the prospect is not played out and paid out in the present, but at some future time $t > 0$, two more factors become important. First, we follow the standard approach and model people's willingness to postpone gratification by a constant rate of time preference $\eta \geq 0$, yielding a discount weight of $\rho(t) = \exp(-\eta t)$. This assumption is not crucial for our results - neither a zero rate of time preference nor genuinely hyperbolic time preferences affect our conclusions. A prospect to be played out and paid out at $t > 0$ is discounted for time in the following standard way:

$$V_0(P) = ([u(x_1) - u(x_2)]\, w(p) + u(x_2))\, \rho(t) \tag{3}$$

Second, and most importantly, survival risk changes the nature of the prospect. Following Halevy (2008) and Walther (2010), let $0 < s \leq 1$ denote the constant per-period probability of prospect survival, i.e. the probability that the decision maker will actually obtain the promised rewards by the end of the period. Then the probability that the allegedly guaranteed payment

[6]The same polynomial also emerges in Safra and Segal (1998)'s approach to constant risk aversion.

x_2 materializes at the end of period t is perceived to be s^t, and the probability of the risky component $x_1 - x_2$ effectively amounts to ps^t. Therefore, the objective two-outcome prospect is subjectively perceived as a three-outcome prospect $\tilde{P} = (x_1, ps^t; x_2, (1-p)s^t; 0)$, where the zero outcome captures that "something may go wrong".[7] With the passage of time, the probability of prospect survival gets progressively scaled down. Therefore, subproportional preferences are a natural framework for studying the effects of time on prospect valuation. At the present, the future prospect is evaluated according to

$$\begin{aligned} V_0\left(\tilde{P}\right) &= \left([u(x_1) - u(x_2)]\, w(ps^t) + u(x_2) w(s^t)\right) \rho(t) \\ &= \left([u(x_1) - u(x_2)]\, \frac{w(ps^t)}{w(s^t)} + u(x_2)\right) w(s^t)\rho(t) \end{aligned} \tag{4}$$

Now suppose that an observer assumes that there is no survival risk, i.e. that $s = 1$, while in fact $s < 1$. Consequently, she infers probability weights $\tilde{w}$ and discount weights $\tilde{\rho}$ from observed behavior on the presumption that the decision maker evaluates the objectively given prospect P. However, in the eye of the decision maker the prospect involves an additional layer of risk. If the observer neglects $s < 1$, she estimates preference parameters according to Equation 3:

$$V_0(\tilde{P}) = \left([u(x_1) - u(x_2)]\, \tilde{w}(p) + u(x_2)\right) \tilde{\rho}(t), \tag{5}$$

interpreting $\tilde{w}$ as true probability weights and $\tilde{\rho}$ as true discount weights, while in fact the weights are distorted by survival risk. Obviously, the measured weights are different from the true ones if $s < 1$. By comparing Equation 4 with Equation 5 we can see that the relationships between true and observed preference parameters are given by

$$\tilde{w}(p) = \frac{w(ps^t)}{w(s^t)} \tag{6}$$

$$\tilde{\rho}(t) = w(s^t)\rho(t) \tag{7}$$

These equations define the central relationships between observed and true underlying probability and discount weights. Concerning the discount weights, this representation is equivalent to Halevy (2008)'s who derives this relationship in the context of Yaari (1987)'s dual theory with a convex probability weighting function. Because $\tilde{w}(p) \neq w(p)$ and $\tilde{\rho}(t) \neq \rho(t)$ for subproportional preferences, survival risk drives a wedge between true underlying preferences and observed risk taking and discounting behavior. Thus, future risk conjointly with proneness to Allais-type behavior provides the missing link between behavior under risk and over time.

[7] We relax this assumption in Section 5.7.

Table 2: Model Variables

	Variable	Description	Characteristics
Prospects	x	monetary payoff	$x \geq 0$
	p	probability of x	$0 \leq p \leq 1$
	s	probability of prospect survival	$s \leq 1$
	$1 - s$	survival risk	
	t	length of time delay	$t \geq 0$
Preferences	$u(x)$	utility function	$u(0) = 0, u' > 0$
	$w(p)$	true probability weight	$w(0) = 0, w(1) = 1, w' > 0$
	η	rate of pure time preference	$\eta \geq 0$, constant
	$\rho(t)$	discount weight	$\rho(t) = \exp(-\eta t)$
Behavior	$\tilde{w}(p)$	observed probability weight	$\tilde{w}(p) = w(ps^t)/w(s^t)$
	$\tilde{\rho}(t)$	observed discount weight	$\tilde{\rho}(t) = w(s^t)\rho(t)$
	$\tilde{\eta}(t)$	observed discount rate	$\tilde{\eta}(t) = -\tilde{\rho}'(t)/\tilde{\rho}(t)$

5 Model Predictions

In the following, we discuss the implications of our approach for the empirical phenomena listed in Table 1 and demonstrate that all of them can be explained by our framework. An important feature of future prospects concerns the timing of the resolution of uncertainty. We distinguish three different cases: First, the prospect is played out and paid out at the same time, i.e. both base risk and survival risk are resolved simultaneously in one shot. This situation is represented by Propositions 1 and 2. Second, both prospect and survival risk are resolved sequentially over the course of time. This case is covered by Proposition 3. Finally, base risk is resolved before the payment date, the topic of Proposition 4. The proofs of the propositions are presented in Appendix B, as is a discussion of the necessity of subproportionality.

5.1 Fact #1: Delay Dependence of Risk Tolerance

Turning to the simultaneous resolution of base risk and survival risk first, we see from Equation 6 that observed probability weights $\tilde{w}(p)$ deviate from true ones $w(p)$ in two respects: First, $w(s^t) < 1$ in the denominator boosts observed weights. Second, $w(ps^t)$ in the numerator distorts observed probability weights. The assumption of subproportional probability weights w generates unambiguous predictions for $\tilde{w}$:

PROPOSITION 1 (*Characteristics of observed probability weights*) Given subproportionality of w and $s < 1$:

1. The function $\tilde{w}$ is a proper probability weighting function, i.e. monotonically increasing in p with $\tilde{w}(0) = 0, \tilde{w}(1) = 1$.
2. $\tilde{w}$ is subproportional.
3. $\tilde{w}$ is more elevated than w. Elevation increases with time delay t and survival risk $1 - s$, at a decreasing rate.
4. $\tilde{w}$ is less elastic than w.
5. The increase in observed risk tolerance is more pronounced for more strongly subproportional risk preferences.

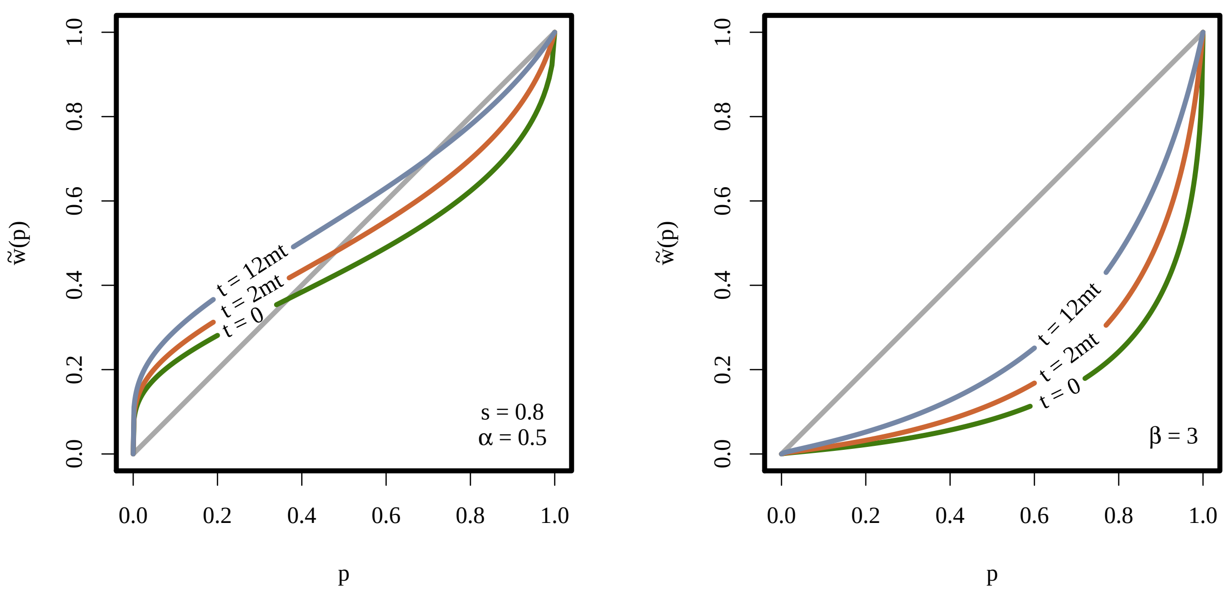
[Proof in Appendix B]

That $\tilde{w}$ is more elevated than w constitutes one of the central implications of our model. Probability weights are larger for delayed prospects and, hence, revealed risk tolerance appears to be higher than risk tolerance for present ones. The departure of $\frac{\tilde{w}(p)}{w(p)} = \frac{w(ps^t)}{w(p)w(s^t)} > 1$ from unity provides a measure of the wedge between observed and true probability weights and corresponds to the strength of the certainty effect inherent in the underlying risk preferences. The intuition behind this result is that, with the passage of time, payoffs become progressively less likely and, therefore, their disappointment potential diminishes commensurately. Since in our model utility from money u is not affected by survival risk, an increase in the elevation of the probability weighting curve gets directly translated into higher revealed risk tolerance. Thus, the presence of future risk makes people appear more risk tolerant for delayed prospects than for present ones. Moreover, $\tilde{w}$ is less strongly curved than w, implying a decreasing proneness to common-ratio violations. Therefore, the risk taking behavior of a typical subject in the lab, where uncertainty resolves almost immediately, is likely to overstate her risk aversion for real-world decisions as well as her proneness to Allais-type violations. Furthermore, the wedge between $\tilde{w}$ and w also increases with the degree of survival risk, implying, somewhat paradoxically, that observed risk tolerance increases with uncertainty.[8] It rises with the degree of subproportionality as well, implying *ceteris paribus* individual-specific sensitivities to delay.

Illustration: Delay Dependence of Probability Weights
The delay dependence of observed probability weights is illustrated in Figure 1. Typically, decision makers exhibit an inverse S-shaped probability weighting function, characterized by underweighting of large probabilities and overweighting of small probabilities of the best outcome. The graph on the left side shows such a decision maker's true probability weights, labeled by $t = 0$,

[8]This finding mirrors Quiggin (2003)'s result of atemporal risk tolerance increasing with background risk.

Figure 1: Effect of Delay on Observed Probability Weights $\tilde{w}$



The graphs show two typical specimens of atemporal risk preferences characterized by the probability weighting curves $w(p) = \tilde{w}(p)$ at $t = 0$: an inverse S-shaped curve on the left and a globally convex curve on the right. Moving the payoff date into the future by $t = 2$ and $t = 12$ months, respectively, shifts the probability weighting curves $\tilde{w}$ upwards. Furthermore, the probability weighting functions get progressively less strongly curved. For purposes of illustration, the curves are derived from Prelec's two-parameter probability weighting function $w(p) = \exp(-\beta(-\ln(p))^{\alpha})$ (Prelec, 1998), assuming degrees of subproportionality $\alpha = 0.5$ and convexity $\beta = 1$ (left hand side) and $\beta = 3$ (right hand side), respectively. Survival risk $1 - s$ is set at 0.2 per annum.

and the respective observed probability weights generated by increasing delay t, depicted by the curves for delays of two and twelve months, respectively. A different specimen of subproportional probability weights is displayed on the right hand side of Figure 1. Here, the probability weighting curve is globally convex, implying pronounced risk aversion.[9]

5.1.1 Evidence of Delay-Dependent Probability Weighting

Empirical evidence on the valuation of delayed prospects typically only provides results on summary measures of risk tolerance. The predominant finding in the literature is higher risk tolerance for delayed prospects than for present ones (Shelley, 1994; Ahlbrecht and Weber, 1997; Sagristano, Trope, and Liberman, 2002; Noussair and Wu, 2006; Coble and Lusk, 2010). Recently, Abdellaoui, Diecidue, and Öncüler (2011) conducted a carefully designed experiment eliciting probability weights for both present and delayed prospects, i.e. in our notation $w(p)$ and $\tilde{w}(p)$. Their results provide persuasive direct support for our approach. They find four distinctive characteristics of delay-dependent prospect valuation. First, the utility for money u does not react to time delay. Second, $\tilde{w}$ is significantly more elevated than w in the aggregate as well as for the majority of the individuals. Third, an additional six-month delay affects elevation less strongly than the first six-month delay. Moreover, $\tilde{w}$ appears to be less strongly curved than w.[10]

5.2 Fact #2: Delay Dependence of Patience

Repercussions on risk taking behavior are not the only effects of survival risk. The same mechanism also affects the valuation of allegedly guaranteed delayed payoffs as it drives a wedge between time preferences and observed discounting behavior. As shown in Equation 7, the observed discount weight for time equals $\tilde{\rho}(t) = w(s^t)\rho(t)$, which depends not only on the *pure* rate of time preference η, but also on the probability of prospect survival s as well as on the shape of the probability weighting function w. Clearly, if w is linear, $\tilde{\rho}$ declines exponentially irrespective of the magnitude of s. To see this, note that $\rho(t) = \exp(-\eta t)$ and $s^t = \exp(-(-\ln(s))t)$, implying a discount rate $\tilde{\eta} = \eta - \ln(s) > \eta$ for $0 < s < 1$. In this case, uncertainty *per se* increases the absolute level of revealed impatience, but cannot account for declining discount rates. Thus,

[9]Convex probability weighting implies risk aversion if utility u is, as usually assumed, (weakly) concave.

[10]In their study on ambiguity, Abdellaoui, Baillon, Placido, and Wakker (2011) show estimates of a probability weighting curve derived from choices over prospects delayed by three months. This curve is also much more elevated than typical atemporal estimates are (see for example Bruhin, Fehr-Duda, and Epper (2010)).

an expected-utility maximizer will exhibit a constant discount rate that is higher than her underlying rate of pure time preference, but her behavior will not show any of the interaction effects addressed in this paper.

If, however, w is subproportional and $0 < s < 1$, the component $w(s^t)$ distorts the discount weight in a predictable way. Increasing patience is not a manifestation of underlying preferences but rather a consequence of survival risk changing the nature of future payoffs. At the level of observed behavior, increasing patience is the mirror image of increasing risk tolerance. In fact, the degree of proneness to common-ratio violations, the degree of subproportionality, can be interpreted as degree of time insensitivity. Intuitively, when the future is inherently risky promised rewards do not materialize with certainty and, therefore, they incorporate the potential of disappointment. Because more immediate payoffs are more likely to actually materialize than more remote payoffs, this potential is perceived to decline with the passage of time and becomes almost negligible for payoffs far out in the future. Technically, since shifting a payoff into the future amounts to scaling down its probability, a decision maker with subproportional preferences becomes progressively insensitive to a given timing difference. These insights are formalized in the following proposition, prediction 3 of which is closely related to Theorem 1 in Halevy (2008). Note that our proposition holds for any subproportional function, and therefore also for inverse S-shaped curves.

PROPOSITION 2 (*Characteristics of observed discounting behavior*) Given subproportionality of w:[11]

1. $\tilde{\rho}(t)$ is a proper discount function for $0 < s \leq 1$, i.e. decreasing in t, converging to zero with $t \to \infty$, and $\tilde{\rho}(0) = 1$.
2. Observed discount rates $\tilde{\eta}(t)$ are higher than the rate of pure time preference η for $s < 1$.
3. Observed discount rates decline with the length of delay for $s < 1$.
4. Greater survival risk generates more strongly declining discount rates.
5. Comparatively more subproportional probability weighting generates comparatively more strongly declining discount rates.

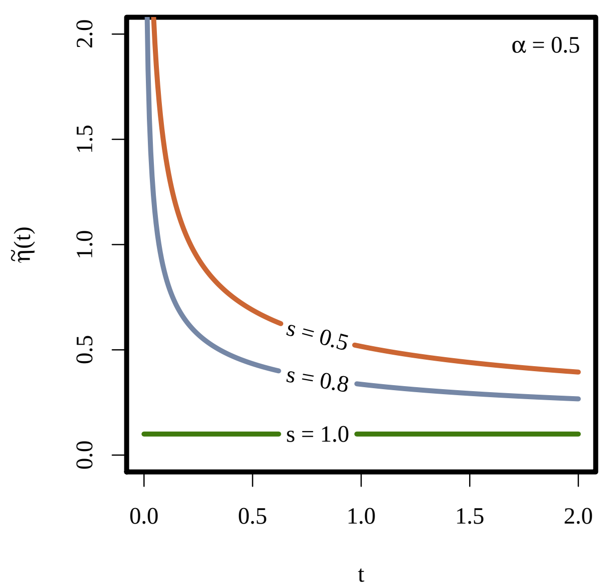
[Proof in Appendix B]

Illustration: Survival Risk and Discount Rates

[11]See Appendix B for a discussion of sufficient versus necessary conditions.

The effects of survival risk on revealed discount rates are presented in Figure 2, which depicts a typical decision maker's observed *discount rates* $\tilde{\eta}$ as they react to varying levels of s. The horizontal line represents the case of no survival risk. In this case, the observed discount rate $\tilde{\eta}$ is constant and coincides with the true underlying rate of time preference η. When survival risk comes into play, however, discount rates decline in a hyperbolic fashion, and depart from constant discounting increasingly strongly with rising uncertainty, as shown by the curves for $s = 0.8$ and $s = 0.5$, respectively.

Figure 2: Effect of Survival Risk on Observed Discount Rates $\tilde{\eta}$



The graph shows discount rates as they move with the length of delay t for different levels of survival risk $1 - s$, where s denotes the probability of prospect survival. When there is no survival risk, $s = 1$, the observed discount rate is constant and equals the rate of pure time preference (line labeled by $s = 1.0$). The higher is the level of risk, the lower s, the more pronounced the hyperbolic decline of discount rates over time is for decision makers with subproportional probability weights (curves labeled by $s = 0.5$ and $s = 0.8$). $\tilde{\eta}(t) := -\frac{\partial \tilde{\rho}}{\partial t} / \tilde{\rho}$. w is specified as Prelec's probability weighting function (in this example $\alpha = 0.5$ and $\beta = 1$).

5.2.1 Evidence of Link between Subproportionality and Hyperbolicity

A large body of empirical evidence documents the prevalence of common-ratio violations as well as of non-exponential discounting, at least at the level of aggregate behavior (Kahneman and Tversky, 1979; Thaler, 1981; Benzion, Rapoport, and Yagil, 1989; Starmer and Sugden, 1989), which suggests that there may be a common cause driving behavior in both decision domains

(Prelec and Loewenstein, 1991). However, there is vast heterogeneity in individuals' behaviors (Hey and Orme, 1994; Chesson and Viscusi, 2000; Bruhin, Fehr-Duda, and Epper, 2010) and the question arises whether common-ratio violations and non-constant discounting are actually exhibited by the same people. Our framework predicts not only the existence of a link between subproportional risk preferences and hyperbolic discounting but also its strength: higher degrees of subproportionality are predicted to be associated with higher degrees of hyperbolicity.

Note that we predict merely a correlation between insensitivities to probabilities and delays, and not a one-to-one correspondence à la Baucells and Heukamp (2012), because our approach allows for many combinations of preferences: For example, an expected utility maximizer may have genuinely hyperbolic time preferences or a probability weigher may not perceive the future as inherently risky and, hence, will exhibit constant discounting. The model can, therefore, pick up a lot of individual heterogeneity, which is beyond the means of Baucells and Heukamp (2012)'s representation. There is another advantage of our structural approach: It defines drivers of behavior that enable the researcher to understand real-world phenomena. Furthermore, these drivers can be experimentally manipulated and their effects tested.

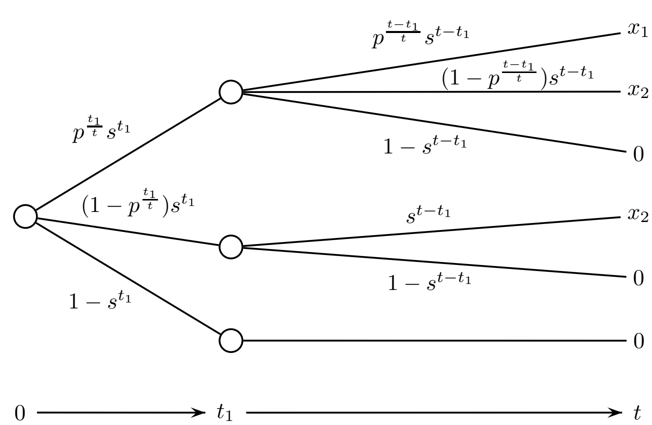
In a recent experimental study, Epper, Fehr-Duda, and Bruhin (2011) provide evidence that subjects' departures from linear probability weighting are indeed highly significantly correlated with the strength of the decrease in discount rates. Moreover, in line with our framework, the curvature of the utility function seems not to be directly related to their decline. In fact, the only variable associated with decreasing discount rates turns out to be the degree of nonlinearity of probability weights, which explains a large percentage of the variation in the extent of the decline, whereas observable individual characteristics, such as gender, age, experience with investment decisions and cognitive abilities are not significantly correlated with the degree of non-constant discounting.

5.3 Facts #3 and #4: Process Dependence

So far, we have considered the case of uncertainty resolving in one single stage, the domain over which atemporal risk preferences are defined. If uncertainty does not resolve in one shot but rather sequentially over the course of time, future prospects lose their single-stage quality and turn into multi-stage ones. In this case the question arises in which way multi-stage prospects are transformed into single-stage ones. In Appendix A we analyze in detail two different transformation methods, reduction by probability calculus and folding back, and their interactions with subproportional risk preferences. One of the issues concerns dynamic consistency. Dynamic consistency requires that choices made at, or plans formed at, different times conform with one another (Sugden, 2004). As Loomes and Sugden (1986) argue, any theory that accommodates the common-ratio effect must dispense either with dynamic consistency or with the compound probability axiom, i.e. reduction by the probability calculus. Therefore, if the decision maker cares only about the total probabilities of the final outcomes she will be dynamically inconsistent unless she precommits herself to stick to her original plans. Folding back, on the other hand, ensures dynamic consistency but, as Propositions 3 and 4 will show, has substantial consequences for revealed risk taking behavior. Therefore, as will become clear, we label adoption of folding back as myopic. In the following, we set $\rho = 1$ for ease of exposition.

Suppose that uncertainty is partially resolved at some future time t_1 and fully resolved at the payment date t, as depicted in Figure 3. Proposition 3 shows that a decision maker with subproportional preferences prefers uncertainty to be resolved at the payment date t rather than at some earlier time. Note that this result does not hold generally under subproportionality in rank-dependent utility but only applies to the class of prospects studied here, i.e. prospects that are devalued by survival risk without effects on the rank order of the outcomes (see Dillenberger (2010) and the discussion in Appendix A).

Figure 3: Gradual Resolution of Uncertainty



PROPOSITION 3 (*Preference for one-shot resolution of uncertainty*) Given subproportionality of w, $s \leq 1$ and folding back:

1. A myopic decision maker prefers one-shot resolution of (total, i.e. base and survival) uncertainty to sequential resolution of uncertainty.
2. Her preference for one-shot resolution declines with (total) probability of the best outcome.
3. Her prospect valuation is lowest at midterm to maturity.

[Proof in Appendix B]

A special case is the valuation of allegedly certain future payoffs, which constitute simple prospects in our framework. A myopic decision maker, applying folding back, will exhibit a discount weight of $w(s^{t_1})w(s^{t-t_1}) < w(s^t)$, an incident of *subadditive discounting* (fact #4).

Preference for one-shot resolution of uncertainty is embodied in the characteristics of atemporal risk preferences and, therefore, all the insights of Segal (1990), who analyzes two-stage prospects in an atemporal setting, still apply. The effect of sequential resolution of uncertainty on subproportional probability weights is depicted in the left panel of Figure 4. The passage of time does not interact with this preference as long as there is no disassociation of base risk from survival risk. However, revealed risk tolerance is additionally influenced by its delay dependence, as shown in the right panel of Figure 4. Consider a prospect with a long time horizon t. If its total uncertainty is resolved in one single stage, risk tolerance as well as the corresponding discount weight attains its maximum value. If uncertainty resolves gradually, both observed risk tolerance and the discount weight are smaller than in the one-shot case. The effect gets more pronounced the finer is the partition of delay t into subintervals. Therefore, anticipating to watch uncertainty resolve over time considerably dampens the effect of long time horizons on observed risk tolerance, because the decision maker is frequently exposed to the possibility of a disappointing outcome. The preference for one-shot resolution is strongest for improbable prospects and declines with rising probability.

Moreover, our model predicts that partitions of the time interval of equal length will be valued particularly unfavorably. Partitions of equal length correspond to the least degenerate multi-stage prospect and can be interpreted as the relatively most ambiguous situation, which is strongly disliked by people with subproportional preferences (Segal, 1987b). Because of this characteristic, Segal proposes to model ambiguity aversion by subproportional risk preferences over two-stage lotteries. A recent paper by Dillenberger and Segal (2014) shows that such an approach has another attractive implication: It is able to solve Machina (2009, 2013)'s paradoxes which involve a number of situations where standard models of ambiguity aversion are unable to capture plausible features of ambiguity attitudes.

Illustration: Process Dependence of Probability Weights

Figure 4 demonstrates the sensitivity of subproportional probability weights to the number of evaluation stages m, resulting from partitions of equal length (the most pronounced case). The more frequently feedback is provided, the more pronounced is the dampening effect on revealed risk tolerance, illustrated for the atemporal case in the left panel of Figure 4. The curve for $m = 1$ represents a typical subproportional probability weighting function when outcomes are evaluated in one shot. If uncertainty resolves in two stages with equal probability rather than in one shot, the prospect is effectively evaluated with the probability weighting curve $m = 2$, which shows more pronounced underweighting. At $m = 12$, the curve looks extremely convex, implying strong risk aversion.

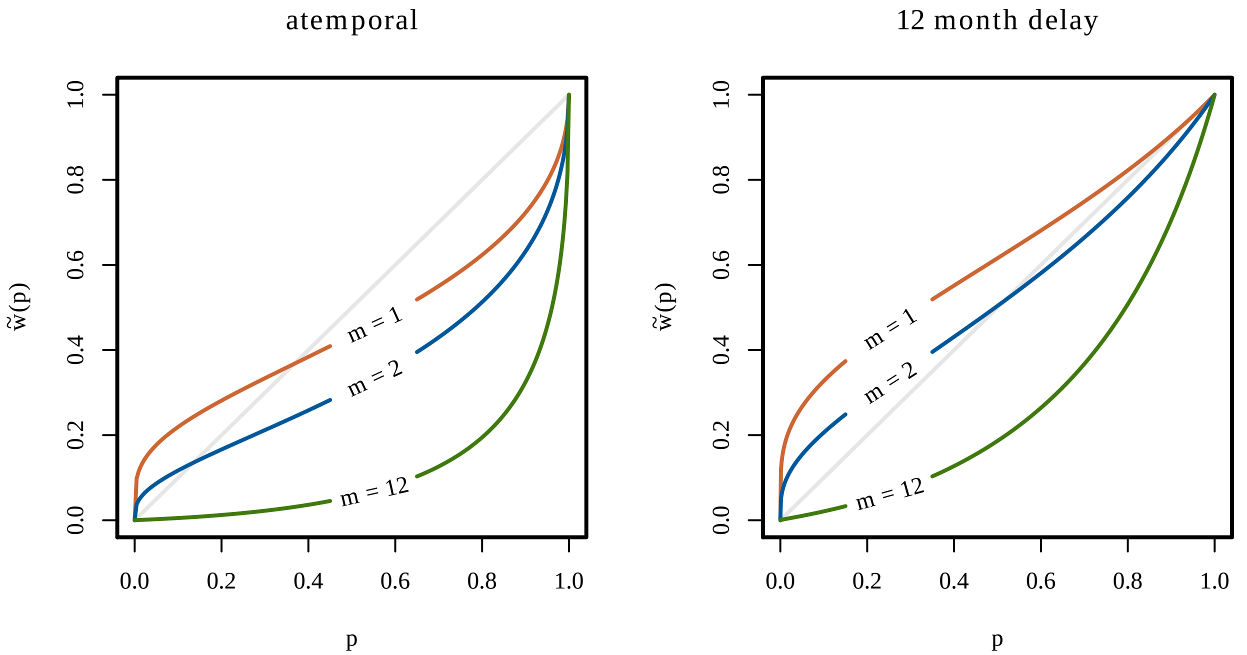
This insight is directly transferable to situations where the resolution of uncertainty involves the passage of real time. The combined effects of survival risk and sequential evaluation are shown in the right panel of Figure 4. When the prospect is delayed by 12 months the curve for $m = 1$ is more elevated than the atemporal curve, a manifestation of delay-dependent risk tolerance. Compounding of weights semi-annually ($m = 2$) or monthly ($m = 12$) looks less dramatic than in the atemporal situation, but shows the same tendency toward convex probability weighting, i.e. an increase in risk aversion relative to the one-shot situation.

5.3.1 Evidence on Sequential versus One-Shot Resolution

A number of prominent papers investigated the effects of feedback frequency and precommitment on people's risk taking behavior in investment games (Gneezy and Potters, 1997; Thaler, Tversky, Kahneman, and Schwartz, 1997; Gneezy, Kapteyn, and Potters, 2003; Bellemare, Krause, Kröger, and Zhang, 2005; Haigh and List, 2005) and generally find that investment behavior appears more risk averse when outcomes are evaluated more frequently.[12] This finding is often interpreted as a manifestation of *myopic loss aversion*, a term coined by Benartzi and Thaler (1995). In this context, myopia is defined as narrow framing of decision situations which focuses on short-term consequences rather than on long-term ones. Loss aversion, one of the key constituents of prospect theory, describes people's tendency to be more sensitive to losses than to

[12] In these experiments subjects evaluate *sequences* of identical two-outcome lotteries over several periods where the range of potential outcomes increases with the number of periods. Unlike Gul (1991)'s disappointment aversion (Palacios-Huerta, 1999; Artsetin-Avidan and Dillenberger, 2011), our model does not deliver clear predictions for this class of prospects.

Figure 4: Process Dependence of Observed Probability Weights



The two panels demonstrate the effect of sequential evaluation on observed probability weights $\tilde{w}$ depending on the number of stages m. The left panel shows atemporal probability weighting functions for one-shot evaluation ($m = 1$) and multi-stage evaluations ($m = 2$ and $m = 12$). The right panel does the same for a 12-month delay under the assumption of an additional layer of risk ($s = 0.8$). When resolution of uncertainty is delayed by 12 months, revealed risk tolerance for $m = 1$ is higher than in the atemporal case. Sequential evaluation ($m = 2, m = 12$), however, has the same qualitative, but less pronounced, effect as in the atemporal model.

gains.[13] According to this interpretation, if people evaluate their portfolios frequently, the probability of observing a loss is much greater than if they do so infrequently. In this sense, myopic loss aversion describes a mechanism similar to sequential probability weighting. Consequently, loss-averse investors shy away from risky assets as probability weighers do.

Several authors have challenged the loss aversion argument, however. Using conventional parameterizations of cumulative prospect theory, Blavatskyy and Pogrebna (2010) show that the effects of myopic loss aversion may be modified by probability weighting and conclude that myopic loss aversion alone cannot explain the observed patterns of behavior. Langer and Weber (2005) argue and support experimentally that, depending on the specific risk profile of the in-

[13]Recently, Köszegi and Rabin (2009) extended the concept of loss aversion to changes in beliefs about present and future consumption. Their model also predicts decision makers to prefer information to be clumped together rather than apart.

vestment sequence, myopia may decrease or increase the attractiveness of a sequence. Regarding the equity premium, De Giorgi and Legg (2012) argue that probability weighting may raise the equity premium considerably above the level predicted by loss aversion alone. The upshot of these arguments is that probability weighting should not be ignored when studying investment behavior. In any case, all the studies on feedback frequency conducted so far have not controlled for subjects' inclinations toward probability distortions and have remained within the confines of atemporal risk preferences.

For the domain of discounting, evidence of process dependence is presented in Read (2001) and Read and Roelofsma (2003). Interestingly, sequential discounting has another implication: People may exhibit hyperbolic discounting when the length of delay is increased, i.e. when uncertainty resolves in a single stage, but constant discounting when two events are shifted into the future by a common timing difference, which may induce folding back. Evidence for the simultaneous occurrence of constant and non-constant discounting is provided by Epper, Fehr-Duda, and Bruhin (2009) and Dohmen, Falk, Huffman, and Sunde (2012).

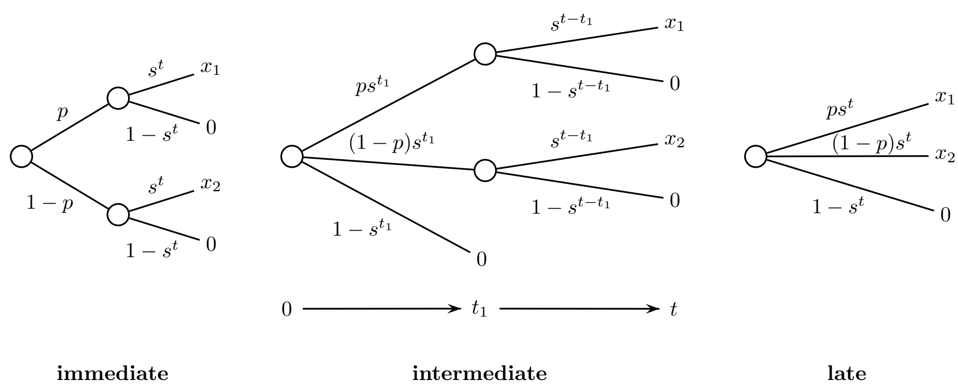
5.4 Fact #5: Timing Dependence of Risk Tolerance

The previous theoretical result rests on the assumption that base risk is resolved simultaneously with survival risk. If the prospect is played out before payment takes place, base risk is segregated from survival risk. As long as base risk is unresolved both types of risks are effective, after resolution of base risk only survival risk remains to be resolved, defining two distinct stages of uncertainty resolution. As Segal (1990) argues, folding back is particularly plausible when sufficiently long time passes between the stages or the stages are clearly distinct.

Figure 5 depicts three different cases: First, the prospect is played out immediately after prospect valuation. In this case, the decision will know the outcome after her decision and faces only survival risk. This situation corresponds to the left panel, labeled "immediate". The right panel shows the other extreme, labeled "late" when the prospect is played out and paid out at the same time t, the focus of Propositions 1 and 2. The middle panel is dedicated to the intermediate case when base risk is resolved at some time t_1 in the future before the payment date.

PROPOSITION 4 (*Preference for late resolution of base risk*) Given subproportionality of w, $s < 1$ and folding back:

Figure 5: Resolution Timing of Base Risk



The figure shows three different timings of the resolution of base risk. Uncertainty gets resolved either immediately at $t = 0$ (left tree), at t_1 between the present and the time of payment (middle tree), or at the time of payment at $t > 0$ (right tree).

1. A myopic decision maker values prospects with base risk resolving at the time of payment more highly than prospects with earlier resolution of base risk.
2. The wedge between late and immediate resolution, $\frac{w(ps^t)}{w(p)w(s^t)}$, declines with probability p.
3. The wedge between late and immediate resolution increases with time horizon t and survival risk $1 - s$.

[Proof in Appendix B]

While it is always the case that late resolution at t is preferred to any earlier resolution time t_1, we cannot ascertain that intermediate resolution at $t_1 > 0$ is generally better than immediate resolution at $t_1 = 0$. Due to the ambiguity effect resulting from time partitions of equal length, discussed above, the discount weight $w(s^{t_1})w(s^{t-t_1})$ decreases with t_1 for $t_1 \in \left[0, \frac{t}{2}\right)$ and increases for $t_1 \in \left(\frac{t}{2}, t\right]$, while risk tolerance increases throughout. Therefore, total prospect value increases with t_1 as long as both factors increase, which is always the case for $t_1 > \frac{t}{2}$. Depending on the relative magnitudes of the effects before $\frac{t}{2}$, prospect value may decrease after $t_1 = 0$ for some time. Obviously, this depends on the prospect under consideration. Table 3 summarizes the effects of resolution timing on observed probability weights $\tilde{w}$ and discount weights $\tilde{\rho}$.

The value of a simple prospect (x, p) amounts to $u(x)w(ps^{t_1})w(s^{t-t_1})$. In this case, the movement of $w(ps^{t_1})w(s^{t-t_1})$ determines the preference for resolution timing. It is straightforward to show that the minimum of the utility weight $w(ps^{t_1})w(s^{t-t_1})$ is attained at $t_1^* = \frac{t}{2} - \frac{\ln(p)}{2\ln(s)}$, which lies below $\frac{t}{2}$. If $t_1^* > 0$, then immediate resolution may be preferred to some later times before $\frac{t}{2}$, otherwise prospect value increases monotonically in resolution time. The latter is the case for $p \leq s^t$. For a given prospect, this condition is more likely to be met for low survival risk and/or short time horizons. The greater the uncertainty or the longer the time horizon, the comparatively less desirable becomes intermediate resolution of uncertainty.

In our view, that atemporal risk preferences induce a preference for late resolution of base risk constitutes the third important insight from our model, besides delay-dependent risk tolerance and hyperbolic discounting. If myopic decision makers perceive the future as inherently risky, this property follows endogenously from subproportionality and does not constitute an independent preference as in the theoretical literature on resolution timing (Kreps and Porteus, 1978; Chew and Epstein, 1989; Grant, Kajii, and Polak, 2000). These theories build on the assumption that the decision maker has an intrinsic preference for early or late resolution of uncertainty and examine the ramifications of this assumption for different models of atemporal risk preferences.

An intrinsic preference for late resolution of uncertainty can also be interpreted as an aversion to non-instrumental information. Information is non-instrumental when no further action can be taken that will change the decision maker's utility.[14] Grant, Kajii, and Polak (1998) present the following example of non-instrumental information:

"Consider, for example, the decision of whether to be tested for an incurable genetic disorder. A director of a genetic counseling program told the New York Times that there are basically two types of people. There are 'want-to-knowers' and there are 'avoiders'. There are some people who, even in the absence of being able to alter outcomes, find information of this sort beneficial. The more they know, the more their anxiety level goes down. But there are others who cope by avoiding, who would rather stay hopeful and optimistic and not have the unanswered question answered." (Grant, Kajii, and Polak (1998), p.234).

That there are different types of decision makers has not only been observed in the context of health-related information but also in the domain of financial prospects, as the following section shows.

[14]There is a number of papers studying preference for instrumental information in non-expected utility models (see for instance Wakker (1988), Schlee (1990), Safra and Sulganik (1995). Li (2011) analyzes aversion to partial information in the context of an ambiguity averse preferences. See also the discussion of the value of information in Dillenberger (2010).

Table 3: Effects of Resolution Timing t_1 on Decision Weights

	Resolution Timing				
	immediate		intermediate		late
	$t_1 = 0$		$0 < t_1 < t$		$t_1 = t$
$\tilde{w}$	$w(p)$	$<$	$\frac{w(ps^{t_1})}{w(s^{t_1})}$	$<$	$\frac{w(ps^t)}{w(s^t)}$
$\tilde{\rho}$	$w(s^t)$	$>$	$w(s^{t_1})w(s^{t-t_1})$	$<$	$w(s^t)$

5.4.1 Evidence of Preference for Late Resolution of Uncertainty

Several experimental studies have investigated people's intrinsic preferences for resolution timing, frequently based on hypothetical questions. The general finding is that there are varying percentages of people with preference for early resolution, preference for late resolution and timing indifference (Chew and Ho, 1994; Ahlbrecht and Weber, 1996; Arai, 1997; Lovallo and Kahneman, 2000; Eliaz and Schotter, 2007; von Gaudecker, van Soest, and Wengström, 2011). However, in line with our predictions, preference for late resolution seems to be particularly pronounced for positively skewed prospects, i.e. for prospects with small probabilities of the best outcome, and increases with time delay. Epstein and Zin (1991) find preference for late resolution of uncertainty in market data on U.S. consumption and asset returns. As in the case for preference for one-shot resolution, none of the studies so far have controlled for nonlinear probability weighting.

5.5 Fact #6: Risk Dependence of Patience

Researchers have been puzzled not only by delay-dependent risk tolerance and preferences with respect to resolution timing but also by other interactions between time and risk, encompassing risk-dependent discounting and diminishing immediacy. As we will show below, these findings can be naturally accommodated within our framework.

Several studies have found that decision makers appear to discount certain future outcomes more heavily than risky ones. Let V_0 denote the *present value* of the prospect $P = (x_1, p; x_2)$ delayed by t periods. Hence, for $\rho = 1$,

$$V_0 = \left([u(x_1) - u(x_2)] \frac{w(ps^t)}{w(s^t)} + u(x_2) \right) w(s^t) \tag{8}$$

Furthermore, let V_t denote the *future value* of P as of t:

$$V_t = [u(x_1) - u(x_2)]\, w(p) + u(x_2). \tag{9}$$

Discounting by $w(s^t)$ yields

$$V_t w(s^t) = ([u(x_1) - u(x_2)]\, w(p) + u(x_2))\, w(s^t). \tag{10}$$

According to standard discounting theory, the present value V_0 should be equal to the discounted value of V_t, namely $V_t w(s^t)$. However, because $w(p) < \frac{w(ps^t)}{w(s^t)}$, actually $V_t w(s^t) < V_0$. Therefore, it seems as if the certain value V_t is discounted more heavily than the (at t equally attractive) future prospect. The difference in the valuations is not caused by different rates of time preference for risky and certain payoffs, however, but by survival risk changing the nature of the future prospect when evaluated from the point of view of the present rather than from the point of view of the future. *Risk-dependent discounting* was found in several studies. Ahlbrecht and Weber (1997) replicated previous results of Stevenson (1992) only in matching tasks, involving elicitation of V_0 and V_t, but not in choice tasks. In their choice tasks, subjects were asked to choose between a prospect to be played at time t and a certain payment at t. Risk-dependent discounting was tested by varying t. The authors surmised that, as time passes, preference for the prospect over the certain payment should become more pronounced, which was not the case in their choice data, however. How can the absence of an effect in choice tasks be rationalized within our framework? When risky and certain prospects are evaluated concurrently only atemporal risk preferences play a role in subjects' elicited choices. Therefore, subjects' preference ordering over risky and certain payments should remain stable when varying t - this is exactly what Ahlbrecht and Weber (1997) found.

The same kind of risk dependence is at work when the revealed preference for a certain smaller present payoff over an allegedly certain larger later payoff decreases substantially when both payoffs are made (objectively) probabilistic. This finding was labeled *diminishing immediacy* (Keren and Roelofsma, 1995; Weber and Chapman, 2005) and motivated Halevy (2008)'s work. Because of the certainty effect, the additional layer of riskiness affects the later payoff much less than the present one because, due to survival risk, it is viewed as a risky prospect already from the outset.

5.6 Fact #7: Order Dependence of Risk Tolerance

An equivalent analysis can be applied to the issue of order dependence of prospect valuation. In principle, there are three different methods of establishing a decision maker's value of a prospect $P = (x_1, p; x_2)$ delayed by t periods: the time-first order, the risk-first order, or the direct method. The time-first order encompasses, at the first stage, the elicitation of the present risky prospect which is considered to be equivalent to the future one and, at the second stage, the elicitation of the certainty equivalent of this present risky prospect. The risk-first order reverses the elicitation stages and assesses the certainty equivalent as of time t first and its present value thereafter. The direct method, finally, elicits the present certainty equivalent of the delayed prospect in one single operation.

When the decision maker is required to state the prospect's value when discounting solely for risk, she ignores the dimension of time and reports V_t, the value of which gets discounted to $V_t w(s^t)$. Conversely, when discounting for time first, she states the present prospect which is equivalent to the delayed one, evaluated as $\left([u(x_1) - u(x_2)] \frac{w(ps^t)}{w(s^t)} + u(x_2)\right) w(s^t)$. Discounting for risk at the second stage results in its value V_0, which is equal to the present value elicited by the direct method.

Therefore, we predict that discounting for risk first results in a lower prospect valuation than discounting for time first. Moreover, discounting for time first is equivalent to prospect evaluation in one single operation. In their study on order dependence, Öncüler and Onay (2009) indeed found this pattern: While valuations resulting from the time-risk order and the direct method are not statistically distinguishable from each other, risk-time evaluations are significantly lower than the ones obtained from the other two methods (see also Ahlbrecht and Weber (1997)).

5.7 Facts # 8 and # 9: Stake Dependence

So far, "something may go wrong" manifests itself as a potentially zero outcome. In the following, we show that the model provides interesting new insights when we relax this assumption. Survival risk may not lead to a total loss of the allegedly certain outcome x_2 but may entail an outcome that is only somewhat worse than x_2. For example, additional transaction costs of collecting future payoffs may arise or payoffs may materialize much later than anticipated.[15] We model this cost here as δx_2 with a (constant) depreciation rate δ, $0 \leq \delta < 1$. Due to survival risk, the lottery $P = (x_1, p; x_2)$ is perceived as $\tilde{P} = (x_1, ps^t; x_2, (1-p)s^t; \delta x_2)$ in this case.[16]

Applying RDU and discounting by $\rho(t)$ renders the present utility of $\tilde{P}$:

$$V_0(\tilde{P}) = \left[(u(x_1) - u(x_2))w(ps^t) + u(x_2)(w(s^t) + \frac{u(\delta x_2)}{u(x_2)}(1 - w(s^t)) \right] \rho(t) \tag{11}$$

Denoting the ratio $\frac{u(\delta x_2)}{u(x_2)}$ by $r = r(x_2; \delta, u)$, yields the following relationships between true and observed weights $\tilde{w}$ and $\tilde{\rho}$:

$$\tilde{w}(p) = \frac{w(ps^t)}{(1-r)w(s^t) + r} \tag{12}$$

$$\tilde{\rho}(t) = [(1-r)w(s^t) + r]\rho(t) \tag{13}$$

Note that $0 \leq \delta \leq r \leq 1$ holds for concave utility functions u. Comparing these expressions with the respective ones of the basic model, Equations 6 and 7, shows that the basic discount factor $w(s^t)$ is a special case of the general factor $(1-r)w(s^t) + r$. As the latter is a convex combination of $w(s^t)$ and 1, $w(s^t) \leq (1-r)w(s^t) + r \leq 1$ holds. Moreover, contrary to the basic model, the general factor depends on outcome magnitude and the characteristics of the utility function. What are the implications of this general framework for risk taking and observed discounting behavior?

Examining the discount weight $\tilde{\rho}$ first, the following results emerge:

PROPOSITION 5 (*Characteristics of observed general discount weights*) Given subproportionality of w, $s < 1$, and $r < 1$:

1. It is straightforward to establish that, as in the basic model, $\tilde{\rho}(t)$ defines a proper discount function.
2. $\tilde{\rho}$ declines hyperbolically for subproportional w, $s < 1$, and $r < 1$. For $r = 1$, $\tilde{\rho} = \rho$. However, as $(1-r)w(s^t) + r \geq w(s^t)$ holds, discount weights are greater than or equal to the basic case.
3. Under non-constant elasticity of the utility function u, discounting depends on stake size.

[15] Onay and Öncüler (2007) show that behavior under timing risk contradicts discounted expected utility but can be explained by nonlinear probability weighting. See also Chesson and Viscusi (2000).

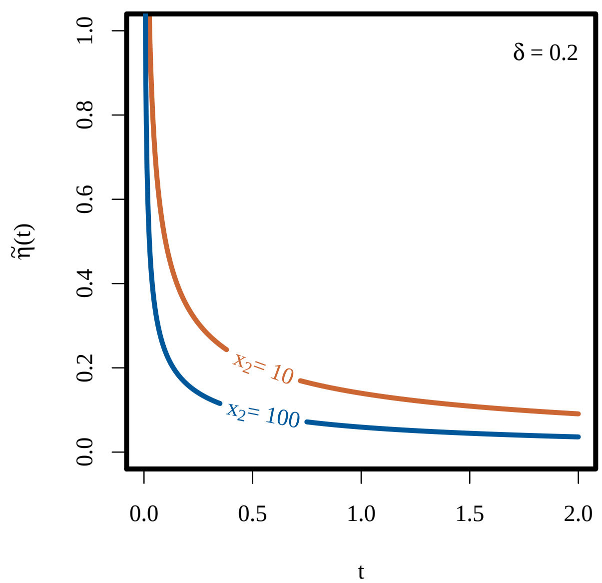
[16] The case of $\delta = 0$ corresponds to the basic model discussed so far, whereas $\delta = 1$ implies that survival risk changes the probability distribution in favor of the smaller outcome x_2.

If the elasticity of u is decreasing, discount weights increase with stake size.

[Proof in Appendix B]

Illustration: The Magnitude Effect
Figure 6 displays the magnitude effect for a decreasing-elasticity utility function. For example, both the exponential utility function and the hybrid expo-power specification display decreasing elasticity under appropriate parameter restrictions. In particular, the expo-power function can accommodate increasing (atemporal) relative risk aversion and decreasing absolute risk aversion at the same time, features that seem desirable in the light of the empirical evidence (Holt and Laury, 2002). Therefore, in our model, a utility function with decreasing elasticity accommodates both atemporal relative risk aversion increasing with stake size and observed impatience decreasing with stake size, the coexistence of which has been puzzling decision researchers for a long time (Prelec and Loewenstein, 1991).

Figure 6: Effect of Stake Size on Observed Discount Rates



The graph shows discount rates as they move with the length of delay t for different levels of the certain outcome x_2, where $u(x) = \ln(x)$ and δ, the rate of depreciation of x_2, is fixed at 0.2. $\tilde{\eta}(t) := -\frac{\partial \tilde{\rho}}{\partial t} / \tilde{\rho}$, with $\tilde{\rho}(t) = [(1-r)w(s^t) + r]\rho(t)$. w is specified as Prelec's probability weighting function (in this example $\alpha = 0.5$ and $\beta = 1$).

Turning to the probability weights, we find the following characteristics:

PROPOSITION 6 (*Characteristics of observed general probability weights*) Given subproportionality of w, $s < 1$, and $r < 1$:

1. $\tilde{w}$ is increasing in p with $\tilde{w}(0) = 0$ and $w(s^t) \leq \tilde{w}(1) \leq 1$. $\tilde{w}(1) = 1$ holds for $r = 0$, i.e. if survival risk is perceived to result in a potentially zero outcome, $\tilde{w}(1) < 1$ otherwise. In the limiting case of $\delta = 1$, $\tilde{w}(1) = w(s^t)$.
2. $\tilde{w}$ is subproportional and less elastic than w.
3. $\tilde{w}$ is more (equally, less) elevated than w depending on the following condition:

$$\tilde{w}(p) - w(p) \gtreqless 0 \quad \text{if} \quad \frac{\frac{w(ps^t)}{1-w(ps^t)}}{\frac{w(p)}{1-w(p)}} \gtreqless r. \tag{14}$$

Illustration: Decreasing Risk Tolerance
If u exhibits decreasing elasticity, observed probability weights decline with x_2, as is evident in Figure 7, thereby increasing observed risk aversion for delayed prospects. In a recent paper, Fehr-Duda, Bruhin, and Epper (2010) indeed find a significant effect of stake size on probability weights. The experiment was conducted in Beijing, China. Subjects had never participated in an incentivized experiment before, which may have induced some uncertainty whether promised payments would actually arrive.

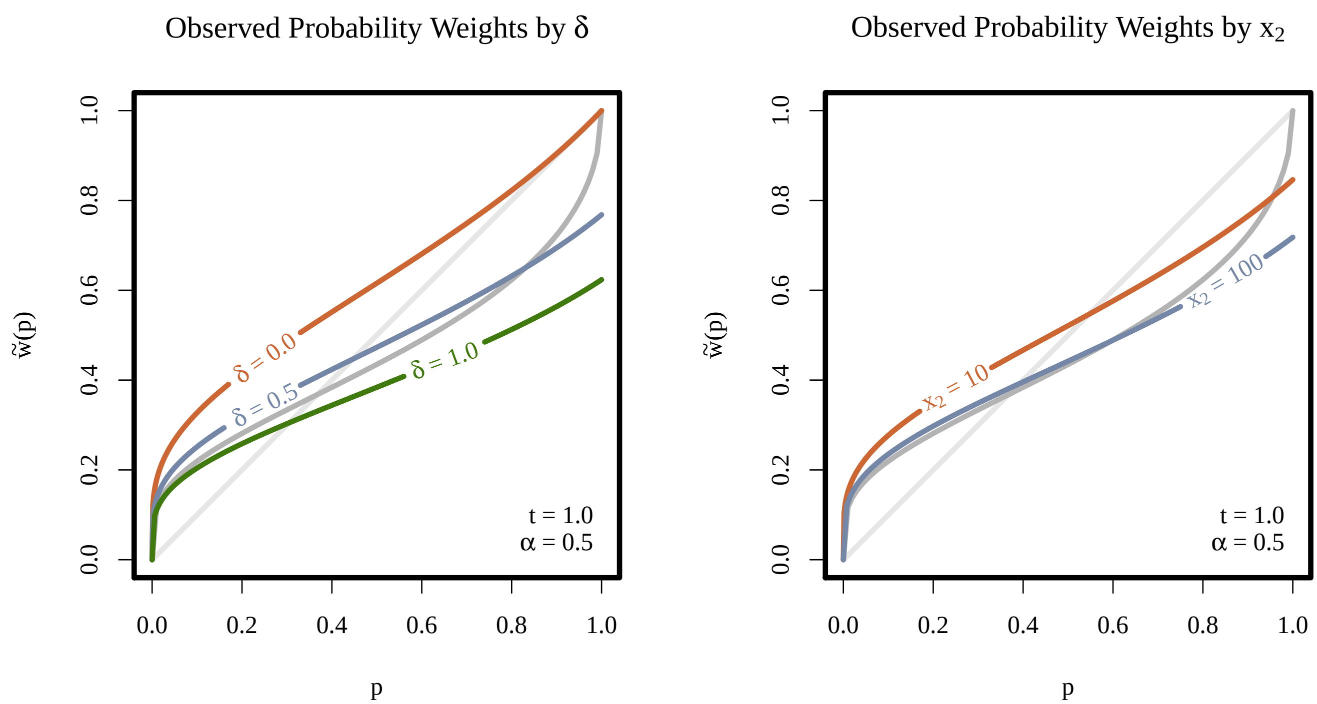
$\tilde{w}$ is reminiscent of Prelec (1998)'s three-parameter probability weighting function $w(p) = \gamma \exp(-\beta(-\ln p)^\alpha)$, where the parameter γ is an index of the weight of certain outcomes relative to uncertain ones. If $\frac{\partial r}{\partial x} = 0$, $\frac{\partial \tilde{w}(1)}{\partial x} = 0$ is satisfied and, given s and t, $\gamma = \frac{1}{w(s^t)(r-1)+r}$ can be interpreted as such a fixed certainty-effect parameter. If, however, $\frac{\partial r}{\partial x}$ is greater than zero, measured probability weights shift downwards with increasing stake size. A summary of the stake effects is provided in Table 4.

For an inverse S-shaped w, $\tilde{w}$ cuts w from above when the ratio in Equation 14 equals r, in other words, over some probability range observed risk taking behavior looks more risk tolerant or less so than in the atemporal case. In the basic model, $r = 0$ and, therefore, $\tilde{w}$ always lies above w.[17] The range over which $\tilde{w}(p) - w(p) > 0$ holds is greater the lower is δ, as is evident in Figure 7. In other words, the decision maker appears comparatively more risk tolerant the higher is the anticipated loss if "something goes wrong".

Stake-dependent discount rates may also be responsible for people's preferences for increasing consumption profiles. If discount rates decline sufficiently strongly with outcome magnitude, a consumption stream $(x_1, x_2, ..., x_n)$ with $x_1 < x_2 < ... < x_n$ may be preferred to $(x_n, x_{n-1}, ..., x_1)$.

[17] Convex subproportional weights may, depending on the value of r, never entail a more risk tolerant range.

Figure 7: Effects of Depreciation Rate and Stake Size on $\tilde{w}$



The left panel demonstrates the effect of the depreciation rate δ on observed probability weights $\tilde{w}(p) = \frac{w(ps^t)}{(1-r)w(s^t)+r}$ with $u(x) = \ln(x)$ and x_2 fixed. The right panel shows the effect of x_2 on $\tilde{w}(p)$ keeping δ constant at 0.2. The time horizon is fixed at $t = 1$, atemporal probability weights correspond to Prelec's function with $\alpha = 0.5$ and $\beta = 1$.

This empirically observed preference for improving sequences has not been explained satisfactorily in the previous literature (Loewenstein and Prelec, 1993; Read and Powell, 2002).

Table 4: Magnitude Effects

	Effect of increasing x_2 on	
Elasticity of u	$\tilde{\rho}$	$\tilde{w}$
constant	no effect	no effect
decreasing	increasing	decreasing
increasing	decreasing	increasing

6 Discussion

Most economically important decisions, may they concern health, wealth, love or education involve a significant interval between the time that the relevant decision must be made and the time that all uncertainty is completely resolved. Therefore, our theoretical models of decision making should be able to handle these situations in a satisfactory way. Mounting evidence of significant interaction effects between time and risk challenge the descriptive validity of the standard models that view discounting for risk and discounting for time as independent operations.

Our approach provides not only a unifying explanation for nine puzzling facts uncovered by experimental research but also a novel view on perplexing real-world behaviors. For example, people buy warranties for household appliances at exorbitant prices but are reluctant to buy adequate health insurance. Similarly, people seem overly risk averse when investing in the stock market but are not willing to buy highly subsidized insurance for natural disasters. These examples suggest that risk tolerance, rather than being a manifestation of stable attitudes, depends on the nature of the decision at hand. The puzzle of seemingly volatile preferences can be easily solved, however, if one accounts adequately for the dimension of time along which real-world decisions typically differ. If people perceive the future as inherently risky our model of subproportional preferences predicts revealed risk tolerance to vary systematically with the timing and the process of uncertainty resolution. The longer the time horizon and the lower the frequency of feedback on uncertainty resolution the comparatively more risk tolerant decision makers will appear to be. Thus, the model provides a wide range of testable predictions that will generate new insights into people's economic behavior.

Since warranties for household appliances are typically rather short-term and products are used on a daily basis, consumers, anticipating their disappointment in the case of breakdown, will be easily persuaded to buy warranties. However, when deciding on health insurance they will be much more risk tolerant because health is anticipated to deteriorate very slowly and often does so imperceptibly for a long time. Similarly, it is hard to predict when natural disasters will actually occur and, therefore, floods and earthquakes are not on people's minds. Stock market investors' time horizons may also be long-term in principle but, contrary to the health and disaster insurance cases, information on portfolio performance is easily accessible and, due to its omnipresence in the news, hard to ignore. Frequent checking of newspapers and news tickers will substantially counteract the otherwise risk-tolerance increasing effect of long investment horizons. Delay- and process-dependent risk tolerance not only affects individuals' welfare but also society at large. People's reluctance to take out insurance for floods and earthquakes, for example, poses serious problems when disaster actually strikes. It is practically impossible for the public authorities to deny assistance once there are identified victims and their stories are publicized in the news (Viscusi, 2010). In the context of climate policy, it takes decades or even centuries until the stock of pollutants will be sufficiently reduced to see any gaugeable effect of society's abatement endeavors. If there is both great uncertainty about the effectiveness of abatement policies and lack of feedback, the risk tolerance of a large percentage of the population may be extremely high and, therefore, it is likely that they are opposed to supporting abatement measures.

The ultimate driver of our results is the Allais-type probability dependence of risk attitudes. In conjunction with survival risk, subproportionality of probability weights not only produces higher risk tolerance for future prospects but all the other interactions between time and risk found in the experimental data. Concerning survival risk, we presume that almost everybody has experienced at some time in their past that something may go wrong. Subproportionality of risk preferences, on the other hand, originates from various psychological processes, such as anticipated emotions, loss aversion in the Köszegi and Rabin (2007) sense or salience-driven prospect valuation. Thus, our model is well grounded in decision makers' psychology and is portable to many different approaches of modeling human behavior.

Appendix A: Dynamic Consistency, Myopia and Sophistication

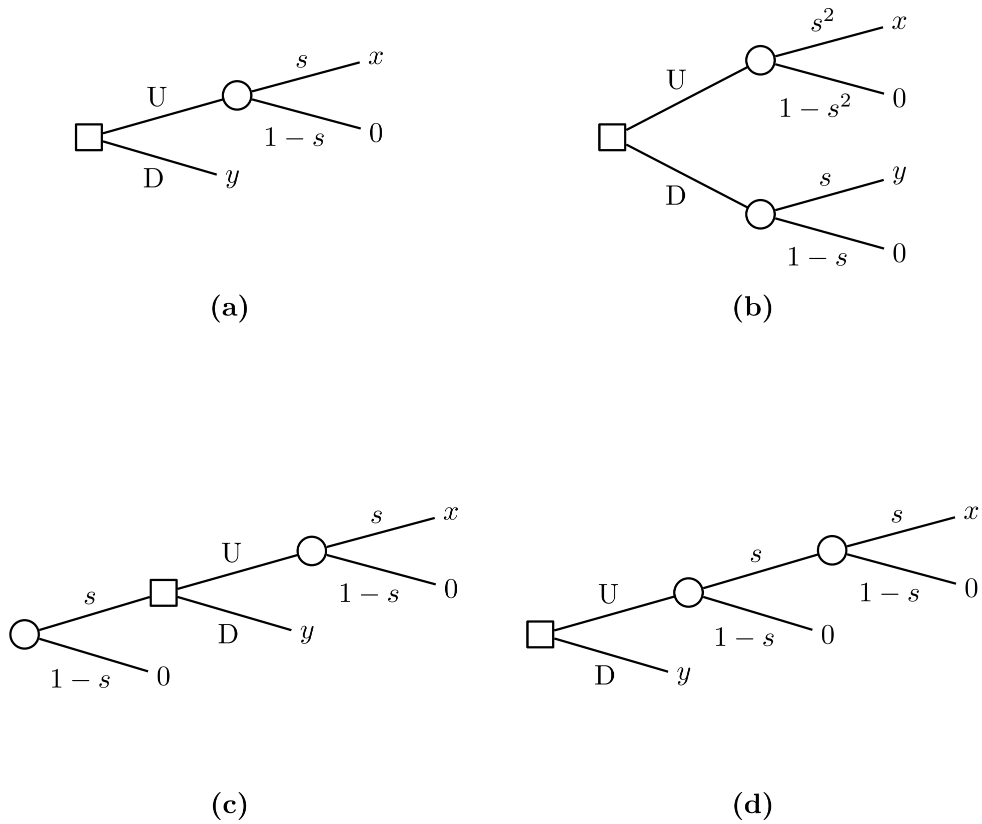
In this section we present background material on the problem of multi-stage prospect valuation, starting with discounting behavior. From a normative point of view, there are two kinds of, logically distinct, problems with hyperbolic discounting. First, hyperbolic discounting violates stationarity, i.e. the requirement that the discount weight over a certain delay remains constant when the delay is shifted into the future. Second, there may be situations in which the decision maker is dynamically inconsistent, i.e. choices made or plans formed at different times do not coincide. In order to take a closer look at these claims and to pave the ground for Propositions 3 and 4, we use the technique of decision trees. Choice nodes are represented by squares and chance nodes are represented by circles. Without loss of generality, we will henceforth assume that the true discount weight ρ equals 1. In the decision trees in Figure 8, the upper branch represents an option U, the lower branch represents an option D. In Figure 8a, the option U entails an, allegedly certain, larger outcome x, to be paid in one period. The option D entails a smaller outcome y, payable immediately. Assume that the decision maker prefers the smaller immediate outcome y, i.e. $u(x)w(s) < u(y)$.

Now suppose that the decision maker faces the choice between x delayed by two periods and y delayed by one period, i.e. both outcomes are shifted into the future by one period. Option U in Figure 8b is associated with a value of $u(x)w(s^2)$ in this case, and option D is associated with $u(y)w(s)$. Due to subproportionality of probability weights w, $(w(s))^2 < w(s^2)$ and hence

$$\frac{u(x)w(s)}{u(y)} = \frac{u(x)w(s)w(s)}{u(y)w(s)} < \frac{u(x)w(s^2)}{u(y)w(s)}. \tag{15}$$

Therefore, the relative value of option U increases and may lead to a change of preference in favor of the larger later outcome x. This type of preference reversal has become known as *common difference effect*, a violation of stationarity, and constitutes one of the most robust empirical findings in intertemporal choice. In the framework of our model, the same mechanism that is responsible for common-ratio violations in risky choice produces violations of stationarity in intertemporal choice if survival risk comes into play. The parallelism between common-ratio violations in atemporal risky choice and violations of stationarity in intertemporal choice was noted by Prelec and Loewenstein (1991).

Figure 8: Static Choice, Dynamic Choice and Precommitment



Tree (a) depicts the choice between an amount x to be paid next period if the prospect survives, and an amount y payable immediately, with $x > y$. The probability of prospect survival is denoted by s. In Tree (b) both options are deferred by one more period. In Tree (c) the decision maker does not decide immediately over the deferred options, but at the end of the first period. Tree (d) represents the case of precommitment.

Let us assume that the common-ratio effect is sufficiently strong such that the decision maker choses option U in this decision situation, i.e. $u(x)w(s^2) > u(y)w(s)$. What happens if the decision maker does not decide now but rather at the end of the first period? This decision situation is depicted in Figure 8c. From the point of view of the present, future uncertainty has to resolve favorably for the options to be still available at the end of the first period. Therefore, the decision maker effectively faces a genuinely dynamic two-stage problem.[18] Since preferences are defined over single-stage risks, multi-stage decision problems have to be transformed into single-stage ones by an appropriate mechanism. An obvious candidate is reduction by the calculus of probability. In this case, the probabilities of reaching the final outcomes are compounded and probability weighting is applied only to the resulting compounded probabilities. This procedure renders $u(x)w(s^2)$ for option U and $u(y)w(s)$ for option D and, therefore, a preference for U. At the end of the first period however, the options are valued as $u(x)w(s)$ and $u(y)$, respectively, leading to a change of plan in favor of D. Unless the decision maker foresees how she will behave in the future and precommits to maintain her original plan of choosing U, she will exhibit dynamically inconsistent behavior. Therefore, revealed behavior over time depends on several factors: the characteristics of atemporal risk preferences, the reduction method, and the use of precommitment. In a recent experiment Halevy (forthcoming) finds that half of his subjects are time consistent, but only two thirds of them exhibit stationary choices. On the other hand, half of the inconsistent subjects display stationary preferences.

In a carefully designed experiment Starmer, Cubitt, and Sugden (1998) show that, in the context of atemporal dynamic risky choices, there is indeed a highly significant difference between behavior in situations with and without precommitment (see also Nebout and Dubois (2014)). In the situation without precommitment the majority of subjects, 71%, choose (in our notation) option D whereas in the (forced) precommitment case, which corresponds to the situation in Figure 8d, only a minority of 43% do so. Hence it seems to make a fundamental difference whether the choice has to be made now or later. This inconsistency constitutes a violation of the principle of timing independence, which requires that at each decision node the decision maker chooses the same path as in the corresponding tree where she precommits to a certain strategy.

Several authors made a case against reduction as an appropriate mechanism of transforming

[18] A decision situation is dynamic if there is at least one chance node preceding a choice node.

multi-stage prospects into single-stage ones. Segal (1990) argues that even if the decision maker accepts the basic laws of probability theory she may have a preference over the number of lotteries she participates in, which invalidates reduction by probability calculus. Segal replaces the reduction axiom by a different axiom, compound independence,[19] which ensures the applicability of folding back as transformation mechanism. Folding back means that a two-stage prospect is evaluated recursively by replacing the second-stage prospect with its certainty equivalent and inserting the utility of the certainty equivalent into the single-stage valuation formula. If the decision maker puts herself into the shoes of her future self facing the decision, she will prefer D just as in the first decision problem and then discount the value of $u(y)$ to the present, which yields $u(y)w(s)$. The present value of option U amounts to $u(x)w(s)w(s)$ in this case. If this method is employed, the decision maker's behavior is consistent in the sense that her preferred option D today will also be the preferred option when she actually decides, i.e. she sticks to her original plan of action.

There is a severe problem with folding back, however. Given the decision maker's preferences in the previous decision situations, the following relationship holds:

$$u(x)w(s)w(s) < u(y)w(s) < u(x)w(s^2), \tag{16}$$

which implies that the decision maker would fare better (in terms of present utility) if she chose U instead of D at the end of the first period, i.e. if she precommitted herself to the plan yielding the compounded final prospect value. Therefore, folding back with subproportional preferences comes at a cost even though it is dynamically consistent. For this reason we will term sequential evaluation of multi-stage prospects by folding back as *myopic* and consistency with compounded final-stage evaluation as *sophisticated*.[20]

The decision situation with precommitment is depicted in Figure 8. Presumably, if the decision is made at the end of the first period rather than immediately, the multi-stage nature of

[19] Let $A = (Z_1, q_1; ...; X, q_i; ...; Z_m, q_m)$ be a two-stage prospect yielding m single-stage prospects Z_j with probabilities q_j, $j \in \{1, ..., i-1, i+1, ..., m\}$, and X with probability q_i, and let $B = (Z_1, q_1; ...; Y, q_i; ...; Z_m, q_m)$ yielding Z_j with probabilities q_j, $j \in \{1, ..., i-1, i+1, ..., m\}$, and Y with probability q_i. Compound independence holds if $A \succsim B \iff (X, 1) \succsim (Y, 1)$ (Segal, 1990).

[20] We do not want to imply that myopia, as defined here, is irrational, however. Loomes and Sugden (1986) argue that "*...people seek consistently to maximize expected satisfaction, where that expectation includes the anticipation of possible disappointment and elation. We cannot see any reason for regarding such a maximand as irrational; nor do we think that any simple experience of satisfaction, whatever its source, can be designated either rational or irrational*" (p.280).

the prospect becomes salient and folding back seems to be a natural, and prima facie perfectly rational, evaluation strategy. If the decision maker evaluates her options by folding back she will still choose D. However, if she integrates the probabilities of survival over the two periods into a single number, i.e. if she employs reduction by probability calculus, she ends up choosing option U. Irrespective of transformation strategy, precommitment serves an important purpose: It either ensures dynamic consistency (reduction) or maximum utility (folding back).

To get an impression of what kind of costs of myopic behavior may be involved consider the following illuminating example discussed by Palacios-Huerta (1999).

Example: The Costs of Sequential Evaluation
"On a given day in June 1994, in Los Angeles, the national soccer teams from Brazil and Italy played in the World Cup final. As most people in the world did, a well-known Brazilian professor of economics in the United States watched the game. After the regulation time the game was tied. After an extra thirty minutes the game remained tied. The soccer champion of the world for the next four years then had to be decided in a five-penalty-kick shoot-out. The professor then switched off his television set, as perhaps did many other people, especially Brazilians and Italians....Why did he do it?" (Palacios-Huerta (1999), p.250). Palacios-Huerta argues that taking the professor through the process of watching the penalty shoot-out increases the number of times that some disappointment may occur and, in this sense the process itself generates a loss of utility, the costs of emotional involvement.[21]

As recent theoretical developments show, nonlinear probability weighting can indeed be rationalized by anticipated emotions of elation and disappointment (Bell, 1982; Gul, 1991; Walther, 2003). For subproportional preferences, $(w(s))^m < w(s^m)$ is implied and, therefore, the difference between $w(s^m)$ and $(w(s))^m$ can be interpreted as an affect premium, the costs of evaluating an m-stage prospect sequentially rather than in one shot.[22] In the soccer example above, the professor avoids these costs by turning off the TV, i.e. by precommitment to be informed only of the final outcome of the shoot-out. In our terminology, he acts in a sophisticated way.[23] If precommitment is possible but costly, the affect premium provides a boundary for the costs of precommitment the decision maker is willing to incur.

In the following we apply this analysis to the valuation of two-outcome risky prospects. Let us abstract from the passage of real time and consider atemporal two-stage problems first. Assume that the prospect $(x_1, p; x_2)$ gets resolved in two stages $((x_1, r; x_2), q; (x_2, 1))$ such that $p = qr$.

[21] A similar reasoning is presented by Loomes and Sugden (1986).
[22] Dillenberger (2010) analyzes this premium in a general context.
[23] For another example of myopia versus sophistication, in the context of casino gambling, see Barberis (2012).

Applying folding back to the two-stage prospect $((x_1, r; x_2), q; (x_2, 1))$ renders a valuation of

$$[u(x_1) - u(x_2)]\, w(q)w(r) + u(x_2). \tag{17}$$

The value of its single-stage counterpart $(x_1, p; x_2) = (x_1, qr; x_2)$ amounts to

$$[u(x_1) - u(x_2)]\, w(qr) + u(x_2). \tag{18}$$

Subproportionality of w implies that $w(qr) > w(q)w(r)$, i.e. one-shot resolution of uncertainty is always preferred to sequential resolution. If sequential resolution is involved, the decision maker looks comparatively more risk averse than in the one-shot case. The difference between sequential and one-shot values, the affect premium, increases with the number of stages m.

The ratio $\frac{w(p)}{w(q)w(r)}$ provides a measure for the strength of the sequential evaluation effect, which exhibits a systematic relationship with respect to probability p (note that q is constant here):

$$\begin{aligned}\frac{\partial\left[\frac{w(p)}{w(p/q)}\right]}{\partial p} &= \frac{w(p)}{pw(p/q)}\left(\frac{w'(p)p}{w(p)} - \frac{w'(p/q)(p/q)}{w(p/q)}\right)\\ &= \frac{w(p)}{pw(p/q)}(\varepsilon_w(p) - \varepsilon_w(p/q))\\ &< 0,\end{aligned} \tag{19}$$

as $p/q > p$ and the elasticity of w is increasing. Therefore, the wedge between one-shot evaluation and sequential evaluation is largest for highly unlikely prospects and decreases with p.

As is clear from Equation 17, it plays no role which stages probabilities q and r are attached to. In this sense, valuation by folding back is symmetrical. However, it makes a difference how total probability p is subdivided. The value of the two-stage prospect attains its minimum for $q = r = \sqrt{p}$, i.e. when the two stages are least degenerate (Segal, 1990). To see this let us examine the derivative of $w(q)w(r)$ w.r.t. r subject to the constraint that $p = qr$:

$$\begin{aligned}\frac{\partial\,[w(q)w(p/q)]}{\partial q} &= w'(q)w(p/q) + w(q)w'(p/q)(-p/q^2)\\ &= 0\end{aligned} \tag{20}$$

$$
\begin{aligned}
&\Rightarrow \quad \frac{w'(q)}{w(q)} q = \frac{w'(r)}{w(r)} r \\
&\Rightarrow \quad q = r = \sqrt{p}
\end{aligned}
\tag{21}
$$

because the elasticity of w is increasing. Therefore, the common-ratio effect observed in single-stage valuations carries over to two-stage prospects.[24] These insights have important implications for our problem of temporal prospect valuation when the resolution timing of the prospect does not coincide with the resolution timing of survival risk or when uncertainty resolves gradually, analyzed in Propositions 3 and 4.

A Note on Sequential Evaluation

In his Proposition 1, Dillenberger (2010) shows that, under recursivity, negative certainty independence (NCI) and a weak preference for one-shot resolution of uncertainty ($PORU$) are equivalent. The NCI axiom requires the following to hold: If a prospect $P = (x_1, r; x_2)$ is weakly preferred to a degenerate prospect $D = (y, 1)$ then mixing both with any other prospect does not result in the mixture of the degenerate prospect D being preferred to the mixture of P. This axiom is weaker than the standard independence axiom and does not put any restrictions on the reverse preference relation when a degenerate prospect is originally preferred to a nondegenerate one. The latter case characterizes the typical Allais common-ratio paradox. NCI allows for Allais-type preference reversals but does not imply them. Dillenberger's Proposition 3 demonstrates that NCI is generally incompatible with rank-dependent utility unless the probability weighting function is linear, i.e. unless RDU collapses to EUT. An intuitive explanation for Dillenberger's Proposition 3 is that under RDU prospect valuation is sensitive to the rank order of the outcomes and, therefore, mixtures with other prospects may affect the original rank order of outcomes in P (and D). How does Dillenberger's result relate to our claim that subproportional probability weights conjointly with recursivity imply a strong preference for one-shot resolution of uncertainty?

The crucial insight is that for the class of prospects studied in this paper changes in rank order do not occur and, hence, NCI is satisfied. To see this, assume that the prospect $(x_1, p; x_2)$, $x_1 > x_2 \geq 0$, gets resolved in two stages $((x_1, r; x_2), q; (x_2, 1))$ such that $p = qr$. In the atemporal case, when there is no additional survival risk, the two-stage prospect continues to be a strictly two-outcome one and the only relevant mixtures are those involving x_2. Suppose that $P = (x_1, r; x_2) \succsim (y, 1) = D$, with $x_1 > y > x_1$ and consider the following mixtures with $(x_2, 1 - \lambda)$ for

[24]Segal (1987b) utilizes this result to explain the Ellsberg Paradox.

some $\lambda \in (0,1)$: $P' = (x_1, \lambda r; x_2)$ and $D' = (y, \lambda; x_2)$. The following relationships hold:

$$\begin{aligned}
P \succsim D \Rightarrow V(P) &= [u(x_1) - u(x_2)]\, w(r) + u(x_2) \geq u(y) \\
V(D') &= u(y)w(\lambda) + u(x_2)(1 - w(\lambda)) \\
&\leq ([u(x_1) - u(x_2)]\, w(r) + u(x_2))\, w(\lambda) + u(x_2)(1 - w(\lambda)) \\
&= [u(x_2) - u(x_1)]\, w(r)w(\lambda) + u(x_2) \\
&< [u(x_2) - u(x_1)]\, w(\lambda r) + u(x_2) \\
&= V(P')
\end{aligned} \tag{22}$$

because $w(r)w(\lambda) < w(\lambda r)$ for any $\lambda \in (0,1)$ (and hence also for $\lambda = q$) due to subproportionality of w. Consequently, for mixtures with the smaller outcome x_2, NCI, and therefore also $PORU$, is *strongly* satisfied. If the mixing prospect may be any arbitrary prospect, in other words if surprises are possible in the course of uncertainty resolution, this result does not hold generally. The only surprise that is still admissible is the occurrence of an outcome worse than x_2, say z. Define $P'' = (x_1, \lambda r; x_2, \lambda(1-r); z)$ and $D'' = (y, \lambda; z)$.

$$\begin{aligned}
V(D'') &= u(y)w(\lambda) + u(z)(1 - w(\lambda)) \\
&\leq ([u(x_1) - u(x_2)]\, w(r) + u(x_2))\, w(\lambda) + u(z)(1 - w(\lambda)) \\
&= [u(x_2) - u(x_1)]\, w(r)w(\lambda) + [u(x_2) - u(z)]\, w(\lambda) + u(z) \\
&< [u(x_2) - u(x_1)]\, w(\lambda r) + [u(x_2) - u(z)]\, w(\lambda) + u(z) \\
&= V(P'')
\end{aligned} \tag{23}$$

For $z = 0$, this case is exactly the situation studied in this paper when survival risk comes into play.

Appendix B: Proofs of Propositions

Proof of Proposition 1

.

1. Since $\tilde{w}(0) = \frac{w(0)}{w(s^t)} = 0$, $\tilde{w}(1) = \frac{w(s^t)}{w(s^t)} = 1$, and $\tilde{w}' = \frac{w'(ps^t)s^t}{w(s^t)} > 0$ hold, $\tilde{w}$ is a proper probability weighting function.

2. Subproportionality of $\tilde{w}$ follows directly from subproportionality of w as for $p > q$:

$$\frac{\tilde{w}(\lambda p)}{\tilde{w}(\lambda q)} = \frac{w(\lambda s^t p)}{w(\lambda s^t q)} < \frac{w(s^t p)}{w(s^t q)} = \frac{\tilde{w}(p)}{\tilde{w}(q)} \tag{24}$$

3. Since w is subproportional,

$$\tilde{w}(p) = \frac{w(ps^t)}{w(s^t)} > \frac{w(ps)}{w(s)} > \frac{w(p)}{w(1)} = w(p) \tag{25}$$

holds for $s < 1$ and $t > 1$. Therefore, $\tilde{w}$ is more elevated than w. Obviously, elevation gets progressively higher with increasing t and an equivalent effect is produced by decreasing s. Since $\tilde{w}$ increases monotonically in t and $\tilde{w} \leq 1$ for any t, elevation increases at a decreasing rate.

4. For the elasticity of $\tilde{w}$, $\varepsilon_{\tilde{w}}(p)$, the following relationship holds:

$$\varepsilon_{\tilde{w}}(p) = \frac{\tilde{w}'(p)p}{\tilde{w}(p)} = \frac{w'(ps^t)ps^t}{w(ps^t)} = \varepsilon_w(ps^t) < \varepsilon_w(p), \tag{26}$$

as the elasticity ε_w is increasing in its argument iff w is subproportional (Segal, 1987a).

5. In order to show that a comparatively more subproportional probability weighting function entails a greater increase in observed risk tolerance we examine the relationship between the underlying atemporal probability weights w and observed ones $\tilde{w}$. Let w_1 and w_2 denote two probability weighting functions, with w_2 exhibiting greater subproportionality. If $w_1(\lambda)w_1(p) = w_1(\lambda pq)$ holds for a probability $q < 1$, then $w_2(\lambda)w_2(p) < w_2(\lambda pq)$ follows as w_2 is more subproportional than w_1 (Prelec, 1998). Choose $r < 1$ such that $w_2(\lambda)w_2(p) = w_2(\lambda pqr)$. For $\lambda = s^t$, the following relationships hold:

$$\frac{\tilde{w}_1(p)}{w_1(p)} = \frac{w_1(\lambda p)}{w_1(\lambda)w_1(p)} = \frac{w_1(\lambda p)}{w_1(\lambda)w_1(p)} \frac{w_1(\lambda)w_1(p)}{w_1(\lambda pq)} = \frac{w_1(\lambda p)}{w_1(\lambda pq)}. \tag{27}$$

Applying the same logic to w_2 yields

$$\frac{\tilde{w}_2(p)}{w_2(p)} = \frac{w_2(\lambda p)}{w_2(\lambda)w_2(p)} = \frac{w_2(\lambda p)}{w_2(\lambda pqr)} > \frac{w_2(\lambda p)}{w_2(\lambda pq)}. \tag{28}$$

Therefore, the relative wedge $\frac{\tilde{w}_2(p)}{w_2(p)}$ caused by subproportionality is larger than the corresponding one for w_1. ■

Proof of Proposition 2

1. $\tilde{\rho}(0) = w(s^0)\rho^0 = 1$. Since $w' > 0$ holds, $\frac{\partial w(s^t)}{\partial t} < 0$ and, therefore, $\tilde{\rho}' < 0$. Finally, $\lim_{t\to\infty} \tilde{\rho}(t) = 0$ (in terms of discount rates: $\lim_{t\to\infty} \tilde{\eta}(t) = \eta$).

2. Discount rates are generally defined as the rates of decline of the respective discount functions, i.e. $\eta = -\frac{\rho'(t)}{\rho(t)}$ and $\tilde{\eta}(t) = -\frac{\tilde{\rho}'(t)}{\tilde{\rho}(t)}$. Therefore,

$$\begin{aligned}
\tilde{\eta}(t) &= -\frac{\tilde{\rho}'(t)}{\tilde{\rho}(t)} \\
&= -\frac{w'(s^t)s^t \ln(s)\exp(-\eta t) - w(s^t)\exp(-\eta t)\eta}{w(s^t)\exp(-\eta t)} \\
&= -\left(\frac{w'(s^t)s^t}{w(s^t)}\ln(s) - \eta\right) \\
&= -\ln(s)\varepsilon_w(s^t) + \eta \\
&> \eta
\end{aligned} \tag{29}$$

   since $\ln(s) < 0, w > 0, w' > 0$. Note that $\frac{w'(s^t)}{w(s^t)}s^t$ corresponds to the elasticity of the probability weighting function w evaluated at s^t, $\varepsilon_w(s^t)$.

3. Since the elasticity of a subproportional function is increasing in its argument, the elasticity of $w(s^t)$ is decreasing in t. Thus,

$$\tilde{\eta}'(t) = -\ln(s)\frac{\partial \varepsilon_w(s^t)}{\partial t} < 0. \tag{30}$$

4. In order to derive the effect of increasing survival risk, i.e. decreasing s, we examine the sensitivity of $\frac{\tilde{\rho}(t+1)}{\tilde{\rho}(t)\tilde{\rho}(1)} = \frac{w(s^{t+1})}{w(s)w(s^t)}$, which measures the departure from constant discounting between periods $t+1$ and t, with respect to changing s:

$$
\begin{aligned}
&\frac{\partial}{\partial s}\left[\frac{w(s^{t+1})}{w(s)w(s^t)}\right]\\
= &\ \frac{1}{[w(s)w(s^t)]^2}\left[(1+t)s^t w(s)w(s^t)w'(s^{t+1}) - ts^{t-1}w(s)w(s^{t+1})w'(s^t) - w(s^t)w(s^{t+1})w'(s)\right]\\
= &\ \frac{1}{s[w(s)w(s^t)]^2}\left[(1+t)s^{t+1} w(s)w(s^t)w'(s^{t+1}) - ts^{t}w(s)w(s^{t+1})w'(s^t) - sw(s^t)w(s^{t+1})w'(s)\right]\\
= &\ \frac{w(s^{t+1})}{sw(s)w(s^t)}\left[\frac{(1+t)s^{t+1}w'(s^{t+1})}{w(s^{t+1})} - \frac{ts^t w'(s^t)}{w(s^t)} - \frac{sw'(s)}{w(s)}\right]\\
= &\ \frac{w(s^{t+1})}{sw(s)w(s^t)}\left[(1+t)\varepsilon_w(s^{t+1}) - t\varepsilon_w(s^t) - \varepsilon_w(s)\right]\\
< &\ 0.
\end{aligned}
$$

As $s^{t+1} < s^t < s$, $\varepsilon_w(s^{t+1}) < \varepsilon_w(s^t) < \varepsilon_w(s)$ and, hence, the sum of the elasticities in the final line of the derivation is negative. Therefore, increasing survival risk, i.e. decreasing s, entails a greater departure from constant discounting and, consequently, a higher degree of hyperbolicity.

5. In order to examine the effect of the degree of subproportionality on hyperbolicity, the strength of decline, suppose that the probability weighting function w_2 is comparatively more subproportional than w_1, as defined in Prelec (1998), and that the following indifference relations hold for two decision makers 1 and 2 at periods 0 and 1:

$$
\begin{aligned}
u_1(y) &= u_1(x)w_1(s)\rho \quad \text{for } 0 < y < x,\\
u_2(y') &= u_2(x')w_2(s)\rho \quad \text{for } 0 < y' < x'\,.
\end{aligned}
\tag{31}
$$

Due to subproportionality, the following relation holds for decision maker 1 in period t:

$$
1 = \frac{u_1(x)w_1(s)\rho}{u_1(y)} < \frac{u_1(x)w_1(s^{t+1})\rho^{t+1}}{u_1(y)w_1(s^t)\rho^t}. \tag{32}
$$

Therefore, the probability of prospect survival has to be reduced by compounding s over an additional time period Δt to re-establish indifference:

$$
u_1(y)w_1(s^t)\rho^t = u_1(x)w_1(s^{t+1+\Delta t})\rho^{t+1}. \tag{33}
$$

It follows from the definition of comparative subproportionality that this adjustment of the survival probability by Δt is not sufficient to re-establish indifference with respect to w_2, i.e.

$$u_2(y')w_2(s^t)\rho^t < u_2(x')w_2(s^{t+1+\Delta t})\rho^{t+1}. \blacksquare \tag{34}$$

Proof of Proposition 3

1. Consider the tree in Figure 3. Here, both base risk and survival risk are assumed to resolve simultaneously in two stages, partially at t_1 and finally at t. Applying folding back, the resulting two-stage prospect is evaluated as

$$[u(x_1) - u(x_2)]\, w(p^{t_1/t}s^{t_1})w(p^{(t-t_1)/t}s^{t-t_1}) + u(x_2)w(s^{t_1})w(s^{t-t_1}). \tag{35}$$

   Subproportionality implies that $w(p^{t_1/t}s^{t_1})w(p^{(t-t_1)/t}s^{t-t_1}) < w(ps^t)$ and $w(s^{t_1})w(s^{t-t_1}) < w(s^t)$.

2. Follows directly from the derivation in Equation 19 in Appendix A.

3. Using the result of the derivation in Equation 20 in Appendix A, both utility weights attain their respective minima at $t_1 = \frac{t}{2}$ when partial probabilities are equal. $\blacksquare$

Proof of Proposition 4

1. Consider the graphs in Fig. 5. The tree on the right-hand side represents the payoff probabilities if base risk is resolved at the time of payment t. As discussed, this prospect is evaluated as $\left([u(x_1) - u(x_2)]\, \frac{w(ps^t)}{w(s^t)} + u(x_2)\right) w(s^t)$. If, however, base risk is resolved immediately after prospect valuation the decision maker will know whether she is supposed to receive x_1 or x_2 at t. Therefore, after resolution of base risk both possible outcomes are only affected by survival risk and get devalued by $w(s^t)$. This situation is shown on the left-hand side of Figure 5. Hence, the value of the prospect immediately before the prospect is played out amounts to

$$\begin{aligned} &[u(x_1) - u(x_2)]\, w(p)w(s^t) + u(x_2)w(s^t) \\ &= ([u(x_1) - u(x_2)]\, w(p) + u(x_2))\, w(s^t). \end{aligned} \tag{36}$$

As $w(ps^t) > w(p)w(s^t)$ is implied by subproportionality of w, prospects with resolution at the date of payment t are valued more highly than prospects with immediate resolution. In fact, in case of immediate resolution of base risk, observed risk tolerance coincides with true risk tolerance and the present value of the prospect is only affected by (hyperbolic) discounting.

What happens if base risk is not resolved immediately but rather at some later time t_1, $0 < t_1 < t$? After t_1, only survival risk remains to be resolved. In this case, the prospect's present value amounts to

$$\left([u(x_1) - u(x_2)] \frac{w(ps^{t_1})}{w(s^{t_1})} + u(x_2) \right) w(s^{t_1}) w(s^{t-t_1}). \tag{37}$$

Subproportionality implies $w(p) < \frac{w(ps^{t_1})}{w(s^{t_1})} < \frac{w(ps^t)}{w(s^t)}$ and, therefore, observed risk tolerance is highest for resolution at payoff time t. Moreover, the late-resolution discount weight $w(s^t) = w(s^{t_1}s^{t-t_1})$ is also greater than $w(s^{t_1})w(s^{t-t_1})$ for any earlier t_1, implying that late resolution is always preferred.

2. Examining the derivative of $\frac{w(ps^t)}{w(p)}$ with respect to p yields

$$\begin{aligned} \frac{\partial \left[\frac{w(ps^t)}{w(p)} \right]}{\partial p} =& \frac{w(ps^t)}{pw(p)} \left(\frac{w'(ps^t)ps^t}{w(ps^t)} - \frac{w'(p)p}{w(p)} \right) \\ =& \frac{w(ps^t)}{pw(p)} (\varepsilon_w(ps^t) - \varepsilon_w(p)) \\ <& 0, \end{aligned} \tag{38}$$

as $p > ps^t$ and the elasticity is increasing. Therefore, the wedge between late evaluation and immediate evaluation decreases with p.

3. The derivative of $\frac{w(ps^t)}{w(s^t)}$ with respect to t yields

$$\begin{aligned} \frac{\partial \left[\frac{w(ps^t)}{w(s^t)} \right]}{\partial t} =& \frac{\ln(s) w(ps^t)}{w(s^t)} \left(\frac{w'(ps^t)ps^t}{w(ps^t)} - \frac{w'(s^t)s^t}{w(s^t)} \right) \\ =& \frac{\ln(s) w(ps^t)}{w(s^t)} (\varepsilon_w(ps^t) - \varepsilon_w(s^t)) \\ >& 0, \end{aligned} \tag{39}$$

as $\ln(s) < 0$, $s^t > ps^t$ and the elasticity is increasing. Therefore, the wedge between late and immediate evaluation increases with time horizon t and, equivalently, with survival risk $1 - s$. ■

Proof of Proposition 5

Items 1 and 2 in Proposition 5 are straightforward to show. Therefore, we only prove item 3. To see that decreasing elasticity of u entails the magnitude effect, we examine the derivative of $\tilde{\rho}$ with respect to monetary outcomes x.

Since $\frac{\partial \tilde{\rho}}{\partial x} = r'(x)(1 - w(s^t))\rho$ holds, the derivative of r is crucial for the direction of stake effects:

$$\begin{aligned}
\frac{\partial r(x)}{\partial x} &= \frac{\partial}{\partial x}\left[\frac{u(\delta x)}{u(x)}\right] \\
&= \frac{u(x)u'(\delta x)\delta - u'(x)u(\delta x)}{[u(x)]^2} \\
&= \frac{u(\delta x)}{xu(x)}\left[\frac{u'(\delta x)\delta x}{u(\delta x)} - \frac{u'(x)x}{u(x)}\right] \\
&= \frac{u(\delta x)}{xu(x)}\left[\varepsilon_u(\delta x) - \varepsilon_u(x)\right],
\end{aligned}$$

where ε_u denotes the elasticity of the utility function.

If the elasticity of u is constant, $r'(x) = 0$ and, therefore, discount weights are independent of stake size. An example of a constant-elasticity function is CRRA power utility. If the elasticity of u is decreasing in x, $r'(x) > 0$ follows and, consequently, discounting behavior exhibits the famous *magnitude effect*, one of the most robust findings in the discounting literature (Frederick, Loewenstein, and O'Donoghue, 2002). ■

Miscellaneous Remarks

On the Necessity of Subproportionality

Clearly, subproportionality is sufficient to produce all the results of Propositions 1 and 2 (as well as of all the following propositions). But is subproportionality, aside from survival risk $1 - s > 0$, also necessary? For statements that refer to the present it is necessary that preferences exhibit the certainty effect, i.e. that $w(p)w(q) < w(pq)$ for any $p, q < 1$, which is implied by but does not imply subproportionality. Therefore, the result that risk tolerance is higher for future prospects than for present ones does not depend on subproportionality, only on the certainty effect. However, for statements pertaining to relationships between behaviors at different times in the future, for instance, that risk tolerance is increasing in t or that discount weights decline hyperbolically, subproportionality is necessary (for a proof with respect to observed discount weights see Saito (2011)). For example, preferences that are not generally subproportional but exhibit the certainty effect, such as the discontinuous weighting function $w(p) = \gamma p$ for $p < 1$

and $w(1) = 1$ defined for $0 < \gamma < 1$, will show an increase in risk tolerance relative to the present as well as quasi-hyperbolic discounting.

Special Cases: Simple and Degenerate Prospects
In our framework a simple prospect (x, p) with one non-zero outcome gets transformed into a prospect (x, ps^t) when it is played out and paid out at t. A degenerate prospect $(x, 1)$ delayed by t is perceived as (x, s^t). Subproportionality implies $\frac{w(1)}{w(s^t)} > \frac{w(p)}{w(ps^t)}$ for $t > 0$ and, therefore, allegedly certain prospects appear to get discounted more heavily than nondegenerate ones, and the effect is more pronounced for low-probability prospects. Note that observations on simple prospects alone do not allow to separate probability weights from discount weights. For this purpose, *nondegenerate two-outcome prospects are needed*.

Application: Constant-Sensitivity Discounting

Ebert and Prelec (2007) argue that time discounting is driven by two distinct forces, impatience and time sensitivity. The authors provide an axiomatic foundation of a constant-sensitivity discount function $\rho(t) = \exp(-(\theta t)^{\alpha})$, where α measures time sensitivity and θ measures impatience.[25] θ marks the boundary between the near and far future: Times shorter than $1/\theta$ are in the near future, while times greater than $1/\theta$ are in the far future. So greater impatience leads to more immediate discounting. The time-sensitivity parameter α, on the other hand, decreases discounting for near-future outcomes and increases discounting for far-future ones. This function fits experimental data remarkably well.

In our framework, the discount function is defined as $\tilde{\rho}(t) = w(s^t)$ for $\rho = 1$. Inserting Prelec (1998)'s specification of the probability weighting function w yields $\tilde{\rho}(t) = \exp(-(\theta t)^{\alpha})$, with impatience defined as $\theta = -\ln(s)$. The subproportionality parameter $\alpha < 1$, therefore, represents an index for time insensitivity: a 1% increase in delay implies an α% reduction in the log-discounted-present-value of the reward. Hence, our model provides a natural link between subproportional probability weighting functions and constant-sensitivity discount functions.

Appendix C: Subproportionality

In this section we review a number of probability weighting functions that are either globally or locally subproportional. We limit our attention to functional forms with at most two parameters. Recall that subproportionality is equivalent to increasing elasticity. Consequently, if the elasticity is U-shaped, the function is superproportional over the range of small probabilities and subproportional over large probabilities. These functions capture the certainty effect but not necessarily general common-ratio violations. Many specifications used in the literature exhibit such a characteristic. Some experimenters found reserve common-ratio violations which require superproportionality over the relevant probability range. Ultimately, it is an empirical issue whether locally or globally subproportional functions fit better.

Polynomials are linear in the parameters and, thus, generally less flexible than specifications that are nonlinear in the parameters. Note that second-order polynomials demarcate the intersection of the class of quadratic utility and RDU.

[25] A flexible specification is presented in Bleichrodt, Rohde, and Wakker (2009). See also Prelec (2004).

Table 5: Probability Weighting Functions

Probability weighting function $\mathbf{w(p)}$	**Parameter range**	**Elasticity**	**Shape**	**Reference**
$\exp(-\beta(-\ln(p))^{\alpha})$	$0 < \alpha < 1$, $\beta > 0$	increasing	inverse S	Prelec (1998)
	$\alpha = 1$, $\beta > 1$	constant	convex	[1]
$\frac{p^{\alpha}}{p^{\alpha}+(1-p)^{\alpha})^{1/\alpha}}$	$0.279 < \alpha < 1$	U-shaped	inverse S	Tversky and Kahneman (1992)
$\frac{\beta p^{\alpha}}{\beta p^{\alpha}+(1-p)^{\alpha}}$	$0 < \alpha < 1$, $\beta > 0$	U-shaped	inverse S	Goldstein and Einhorn (1987)
	$0 < \alpha < 1$, $\beta = 1$			Karmarkar (1979)
	$\alpha = 1$, $\beta < 1$	increasing	convex	Rachlin, Raineri, and Cross (1991)
				Bordalo, Gennaioli, and Shleifer (2012)
$\frac{p+\alpha p(1-p)}{1+(\alpha+\beta)p(1-p)}$	$\alpha > 0$, $\beta > 0$	U-shaped	inverse S	Walther (2003)
$\beta^{1-\alpha}$	if $0 \leq p \leq \beta$: $0 < \alpha, \beta < 1$	constant		[2]
$1-(1-\beta)^{1-\alpha}(1-p)^{\alpha}$	if $\beta < p \leq 1$: -	increasing	inverse S	Abdellaoui, l'Haridon, and Zank (2010)
$\frac{1}{1+\alpha(1-p)}$	$\alpha > 0$	increasing	convex	Gul (1991)
$p-\alpha p+\alpha p^2$	$0 < \alpha < 1$	increasing	convex	Masatlioglu and Raymond (2014); Delquié and Cillo (2006); Safra and Segal (1998)[3]
$p+\frac{3-3\beta}{\alpha^2-\alpha+1}(\alpha p-(\alpha+1)p^2+p^3)$	$0 < \alpha, \beta < 1$	U-shaped	inverse S	Rieger and Wang (2006)
$p-\alpha p(1-p)+\beta p(1-p)(1-2p)$	α depends on β	variety	variety	Blavatskyy (2014)[4]

(1) Equivalent to power specification $w(p) = p^{\beta}$.

(2) For $\alpha > 1, \beta = 1$ constant elasticity, convex; for $\alpha < 1, \beta = 0$ increasing elasticity, convex.

(3) Special case of Blavatskyy (2014) with $\beta = 0$.

(4) Specific parameter constellations with $\beta > 0$ generate inverse S with U-shaped elasticity.

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